FINAL TECHNICAL REPORT

INSTRUMENTATION TECHNIQUES FOR TRACKING LOW-FLYING VEHICLES

EES/GIT PROJECT A-1678

Prepared for
UNITED STATES ARMY
WHITE SANDS MISSILE RANGE
NEW MEXICO 88002

Under

CONTRACT DAAD07-75-C-0025

By

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R. D. Wetherington, E. K. Reedy,
and R. D. Hayes

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ENGINEERING EXPERIMENT STATION
Georgia Institute of Technology
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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(1) 10 feet in position, any axis,
(2) 5 feet per second in velocity, and
(3) 5 feet per second^2 in acceleration, → over

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Both millimeter and laser airborne radars were evaluated as candidates for device development programs, to perform the function of airborne tracking.

The possibility was examined of using an available Ku band airborne radar to determine altitude with 10 foot accuracy, the assumption being that higher horizontal position errors ($\approx$ 50 feet) could be tolerated.
approx

A ground based laser radar network, and a multilateration technique were analyzed. The latter would require range measurements from ground sites to the low-flying target, from the ground sites to an overflying aircraft, and from the aircraft to the low-flying target.

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The results of the analyses indicated that a feasibility study of the multilateration technique should be performed. If more detailed analyses support our findings, a prototype, abbreviated system should be developed, implemented, and tested; and if tests prove the system feasible it should be installed by WSMR.

In the event that the multilateration technique proves to be infeasible (because of excessive terrain masking or multipath effects, for example) it has been recommended that a prototype airborne millimeter or laser tracking radar be developed, be incorporated in a commercially available radio reference system, and then be tested with a minimum of four ground sites.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

There is at present no capability for determining the position, velocity, and acceleration of low flying targets with the accuracy which is increasingly being requested by users of White Sands Missile Range (WSMR). A recent survey of requests for tests (see Appendix A) shows that out of twenty-nine tests there were twenty that were to be flown between zero and two thousand feet above ground level. Six of the twenty-nine users requested position measurement to within five to ten feet, but eleven requested accuracies of one to five feet. The requests for accuracy of velocity measurements for twelve of the programs were one to five feet per second, and for six of the programs velocity accuracy was specified as one-half inch to one foot per second. Similar stringent requests were made for accuracies in acceleration measurements. They ranged from one inch to ten feet per second per second.

The need for measurements on low flying targets is not new. Studies at White Sands Missile Range in 1967 and 1970 showed that the (unattainable) measurement accuracy requirements have been at a consistent level for nearly ten years. The following were suggested (see Appendix B) as realistic design goals for testing low-flying vehicles:

Coverage - 200 feet above ground level

Position measurement accuracy - 10 feet, any axis

Velocity measurement accuracy - 5 feet per second, any axis

Acceleration measurement accuracy - 5 feet per second per second, any axis

Data output - digital format compatible with WSMR telemetry and computer equipment.

It should be noted that these requirements do not satisfy the most stringent requests cited above. They do, however, represent values that are deemed realizable, or almost realizable, with today's technology. Appendix C details the set of constraints and requirements of a WSMR measurement system for testing low-flying missiles and aircraft.

A specific measurement system configuration was suggested by WSMR (Appendix B). As shown in Figure 1, it would include an airborne radar from which to measure the position, velocity, and acceleration of a low-

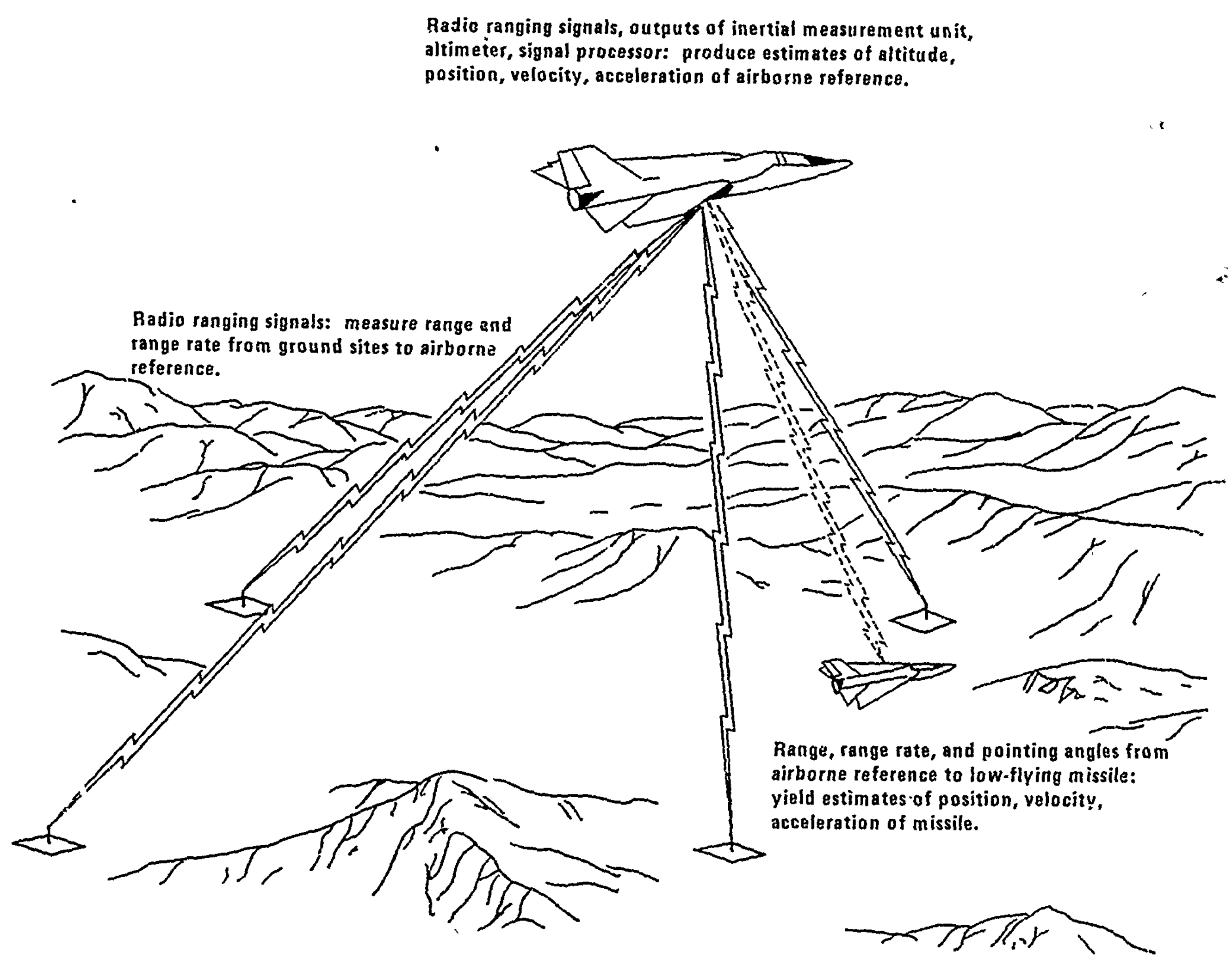

Figure 1. Overflying Aircraft Tracking a Low-Flying Missile.

flying vehicle. The position, velocity, and acceleration of the airborne reference platform would be determined by range and range rate measurements from accurately surveyed ground based stations. The range and range rate signals would be combined with platform attitude signals derived from an inertial navigation unit (INU), and with an altitude signal from a barometric altimeter, to establish the position, velocity, acceleration, and attitude of the airborne reference platform with respect to fixed, ground reference coordinates.

The airborne platform would fly at an altitude sufficiently high to be located by a small number of ground based units, and to permit a subsonic aircraft to track a supersonic vehicle over the length of the range--about one hundred and twenty miles. It was suggested that a small, phased-array radar antenna in the aircraft could track a transponder mounted on the low-flying test vehicle. For system flexibility, the airborne equipment would be mounted in a pod that could be attached to standard Air Force pod hangers. The pod could be a cylinder as large as three feet in diameter and fifteen feet long (Appendix C). The weight budget could be 1000 pounds. The equipment in the pod would include (1) range and range rate measuring devices, (2) an inertial measurement unit, (3) an altimeter, (4) a small digital computer, and (5) a tracking radar. The first four components are found in an existing system, CIRIS (for Completely Integrated Reference Instrumentation System) [1-4].

The purpose of the Georgia Tech study has been to determine:

(1) Will the system of Figure 1, using existing commercial or military equipment mounted as described above, enable White Sands Missile Range to measure the performance of low-flying missiles or aircraft, which have speeds ranging from subsonic to supersonic?

(2) If the system of Figure 1 will not meet the specified accuracy requirements in all axes and over a subsonic to supersonic range of speeds, will it meet the required measurement accuracy in the vertical axis, at subsonic speeds?

(3) Is it feasible to modify the system of Figure 1 to meet the required measurement accuracies?

The following sections of this report detail the findings of the Georgia Tech study, which can be summarized:

(1) Existing airborne equipment was sought to implement the system of Figure 1 so as to meet the accuracy requirements in all axes. Chapter 2 details the search. The CIRIS type of system (see Appendix D) which includes the ground based transponders of Figure 1, and all of the airborne components except the tracking radar, is available and error analysis indicates the system is capable of locating the tracking aircraft with an error of about 5.8 feet, any axis. No airborne radar, for the final link between the low-flying target and the tracking aircraft, was found that would meet the space constraints and the accuracy requirements as defined by WSMR.

(2) The most critical component of the system of Figure 1 is the airborne radar or other sensor which would track the low-flying test vehicle. When no existing airborne radar was found that would meet all requirements, studies were made to determine the feasibility of developing an airborne radar. Alternative approaches, which depart in varying degrees from the Figure 1 concept, were also examined. System parameters for conventional and laser airborne radars are developed in Chapter 3. Three alternatives to the Figure 1 system are described in Chapter 4. The alternatives are:

A ground based network of laser radars.

A Ku band radar in the tracking aircraft which would determine altitude of the test vehicle within 10 feet RMS, but horizontal position error would exceed 40 feet RMS in each horizontal axis.

Multilateration, with radio ranging to the low-altitude target and to the high-altitude aircraft from ground based stations. Range would also be measured to the low-flying target from the high-altitude aircraft.

A fourth multilateration system that would have utilized three CIRIS type airborne reference platforms was discarded because of anticipated operational and scheduling difficulties.

Attention is focused on position error in the following studies, rather than velocity and acceleration errors. This was justified (for the Chapter 3 treatment) by observing that in an analysis of CIRIS [1], velocity errors were two orders of magnitude smaller than position errors. In the Chapter 4 analyses, time did not permit an extension to velocity and acceleration errors.

2. THE SEARCH FOR EXISTING MEASUREMENT SYSTEMS AND EQUIPMENT

In the search for existing systems and equipment which would permit White Sands Missile Range to measure the performance of low-flying vehicles, emphasis was placed on the approach embodied in Figure 1. The functions of the components of the system [1-6] can be outlined:

(1) A radio ranging system (RRS) measures range and range rate from surveyed ground sites to the tracking aircraft.

(2) An airborne inertial measurement unit (IMU) measures acceleration and velocity of its three inertial axes.

(3) A computer in the aircraft combines the RRS and IMU measurements and computes an estimate of position, velocity, and acceleration of the airborne reference coordinates, with respect to the coordinates of the ground sites.

(4) An airborne subsystem measures range and angular directions from the airborne reference coordinates to the low-flying vehicle under test.

(5) A transponder on the low-flying vehicle enhances its "visibility" by increasing the signal-to-noise ratio of the return signal.

The search consisted of a review of reports, papers, and text books on CIRIS type systems [1-6], radio ranging systems (RRS) [6-14], inertial measurement units (IMU) [15-17], barometric altimeters [18], Kalman filter [19-23], airborne radar [24-27], phased array antennas [26, 28-31], atmospheric refraction [1,2,24,32-34], and radar transponders [24-27,35]. Trips were made to the manufacturers of radio position location systems and to Yuma Proving Grounds and White Sands Missile Range to discuss the capabilities and limitations of existing equipment.

Analyses indicated that the constraint on the dimensions of the antenna, which would have to fit within a pod 3 feet in diameter and 15 feet long, and the accuracy requirements ruled out existing airborne radars. It was, however, concluded that CIRIS, which includes the RRS, IMU, barometric altimeter, and airborne computer with Kalman filter, could determine the location of the tracking aircraft with sufficient accuracy if a large enough number of ground based transponders (18) is used.

CIRIS

A version of CIRIS has been assembled and test flown at Holloman Air Force Base. It is reported [36] to be consistently meeting its design goals, which were [1]:

Position: 12.5 feet RMS vector error

Velocity: 0.05 feet/sec RMS vector error

Attitude: 26 $\widehat{\text{sec}}$ RMS vector error*

The geographic coverage, using four ground based transponders, was simulated in a very thorough computer analysis [1] as an isosceles trapezoid measuring 150 and 100 nautical miles on the two parallel sides, and 90 nautical miles on each of the other two. A race track course for an aircraft at 30,000 feet, flying between the two sites separated by 150 nautical miles, was simulated. Using the most optimistic results under the assumption that it represents the best use of measured data by the CIRIS system, it is found that the position error would be about 14 feet, RMS vector, when the aircraft is near one of the transponders. This result can serve as a starting point for estimating errors in a CIRIS type system for the WSMR mission.

Decreasing the distance between the ground based transponder sites would reduce the effect of range scale factor error (which is caused by atmospheric refraction), and the effect of geometric dilution of precision (GDOP). An adjustment can be made to the CIRIS simulation data to account for such a reduction of distance between ground based transponders.

The CIRIS simulation assumed an error of 10 ppm in the survey of its four ground sites, but WSMR survey accuracies approach 2 ppm [37]. The results of the simulation can also be adjusted to reflect the higher survey accuracies.

There is in the literature a considerable range in estimates of the errors caused by multipath and equipment delays. The CIRIS simulation used 3 feet RMS; another report [9] used 6.9 feet RMS, which is probably valid for low (below 10 degrees) elevation angles.

* $\widehat{\text{sec}}$ = arc second

Reducing the CIRIS vector error of 14 feet to account for a ten-to-one reduction in maximum range and a five-to-one increase in survey accuracy, but increasing the error to allow for the more pessimistic estimate of 6.9 feet equipment and multipath errors, the new vector error for radio ranging with transponders about 15 miles apart would be 10 feet RMS, or 5.8 feet RMS per axis.

If the WSMR system is based on a network of squares with a ground site transponder at each corner (see Figure 2) and if the side of each square (80,000 feet) is twice the height of the aircraft above ground level (40,000 feet), the GDOP* factors in vertical and horizontal axes as the aircraft crosses over the center of the square would be the minimum value possible, 0.867 [6]. Low GDOP factors could be maintained as the aircraft approaches the midpoint of a side that is common to two squares if six rather than four transponders are queried. At the crossover point, the GDOP factors would be 0.61 (vertical) and 0.775 (horizontal). For a complete coverage of the 30 by 120 mile range with the 80,000 foot squares, 27 ground based transponders would be required; but only 18 would be required for a corridor as shown in Figure 2. It appears that the CIRIS system now operating at Holloman AFB should be capable of determining the position of an airborne reference flying at 40,000 feet, with a vector error less than 10 feet RMS, or 5.8 feet per axis, with the transponder configuration of Figure 2.

The reference platform position error of 10 feet RMS vector, ascribed here to CIRIS, will be used in the following chapter to determine the allowable error budget for the link from the tracking aircraft to the low-flying target, as conceptualized in Figure 1. There are a number of position locating systems that are similar to CIRIS. These are described in Appendix D. Only CIRIS was (1) designed specifically to effect the location of an airborne instrumentation reference platform, with accuracies approaching the needs of WSMR, and (2) completed and successfully tested.

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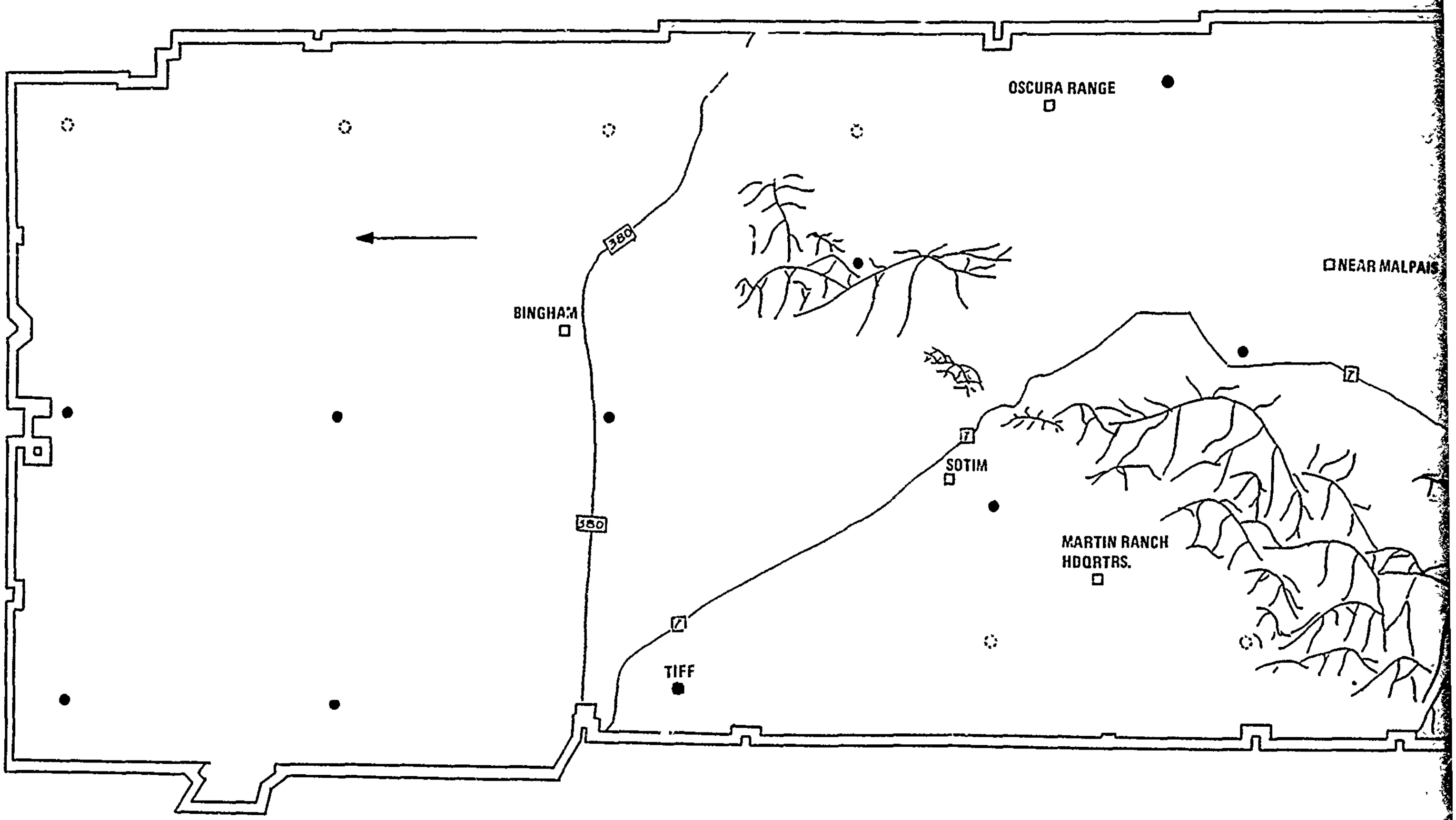
* GDOP = geometrical dilution of precision



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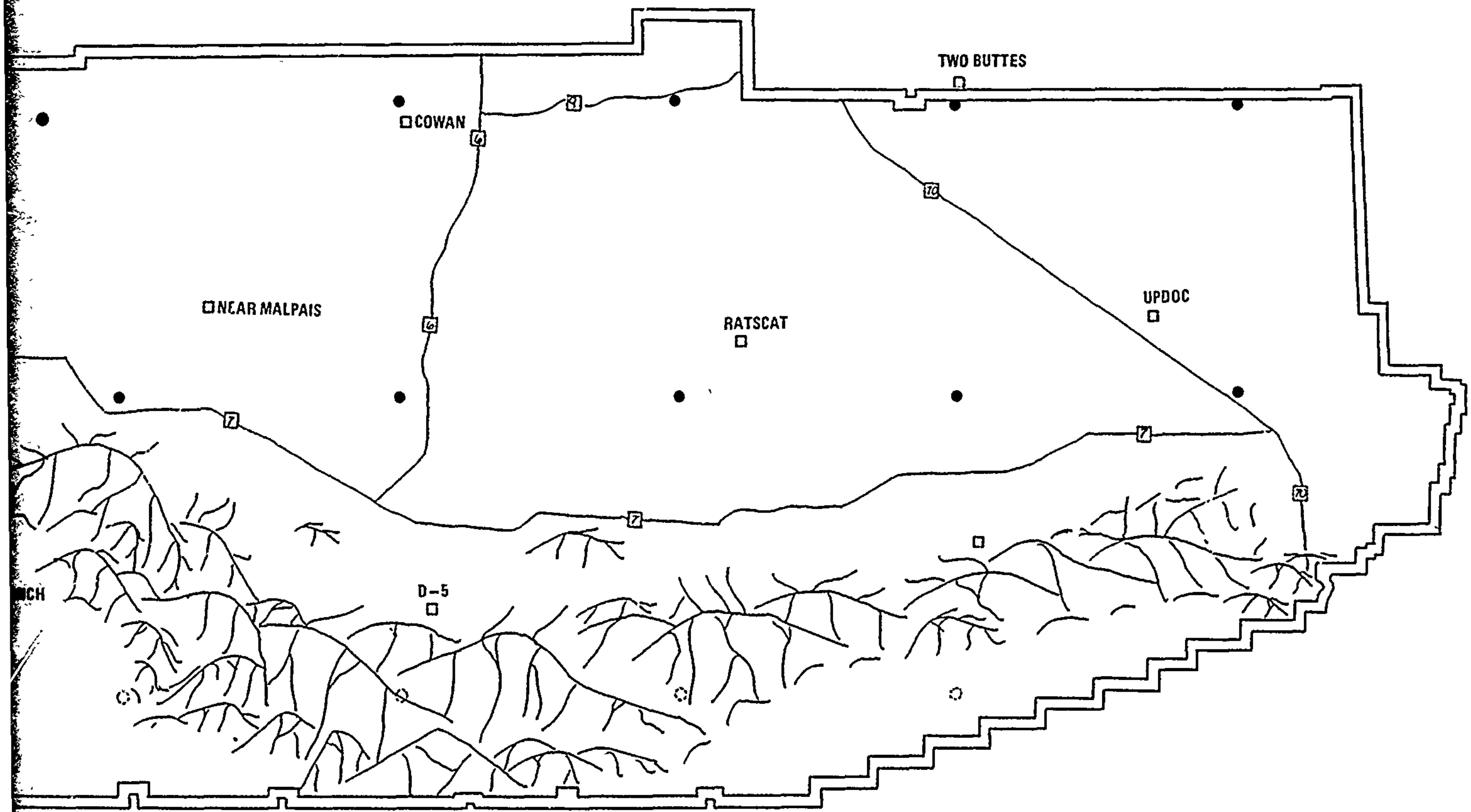

Figure 2. WSMR Ground Sites for Transponders in a CIRIS-Type System.

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3. DESIGN GOALS FOR AIRBORNE RADAR

When no existing radar system or equipment was found which would meet the system constraints and the WSMR accuracy requirements (detailed in Appendix A), a series of studies was initiated to determine the parameters needed for such a radar system. The feasibility of airborne conventional radar and laser radar systems are examined in this chapter. The analysis does not attempt to be rigorous but only seeks to determine feasibility, using deliberately conservative estimates of error.

Assuming that the WSMR position error requirement, stated as "10 feet in any axis" is RMS error, the total allowable vector error would be 17.3 feet RMS. It was estimated in Chapter 2 that using the CIRIS radio ranging and IMU system and the transponder configuration of Figure 2 the error in the position of an airborne reference would be 10 feet RMS vector. The vector error budget for the design goals of a radar tracker would then be

$$\sigma = \sqrt{(17.3)^2 - (10)^2}$$

$= 14.1$ feet RMS, vector error budget for aircraft-to-target tracking system.

where

17.3 feet = vector error budget, target position

10 feet = vector error budget, tracking aircraft position

The tracking error budget for the airborne tracking system in each of three axes is

$$\sigma_T = 14.1/\sqrt{3}$$

$= 8.14$ feet RMS, each axis.

The sources of error in tracking from the airborne reference include:

(1) Uncertainty in the reference system attitude.

(2) Residual range and pointing angle errors arising from atmospheric refraction (after correction is made for meteorological conditions).

(3) Fluctuation range and pointing angle errors caused by local variations in atmospheric refraction.

(4) Radar resolution and equipment errors.

From the CIRIS simulation [1], the attitude uncertainty may be estimated (after making adjustments for smaller distances) as 22.8 $\widehat{\sec}$ or 0.11 milliradian RMS vector. This could be budgeted as 0.078 milliradians in each of two pointing axes:

$$\sigma_{A1} = \sigma_{B1} = \sqrt{1/2\ (0.11)^2} \text{ milliradian}$$
$$= 0.078 \text{ milliradian}$$

The convention adopted here is illustrated in Figure 3. The angle, A, measures the orientation of a vertical plane rotated about the vertical reference axis, Z, of the airborne platform, and A is measured from a reference axis which is independent of the aircraft heading (for convenience it could be true north). The angle B is measured from the true vertical downward direction, which is also independent of the aircraft attitude. The radar antenna needed for the conceptual system of Figure 1 could be assumed to use electrical corrective signals from the CIRIS system which decouple the aircraft's motion. Range is measured radially as in Figure 3 along the line-of-sight.

There is an error in the angle B caused by atmospheric refractivity, N_s (N_s is about 315). If uncorrected, the bias error is [24]:

$$\sigma_{B2}(\text{uncorrected}) = N_s \cot (90 - B)\ \mu\text{rad}$$
$$= N_s \tan B\ \mu\text{rad}$$

If the angle B is restricted to be less than 55 degrees from antenna boresight, the maximum uncorrected error for B is 1.43 N_s microradians. It is

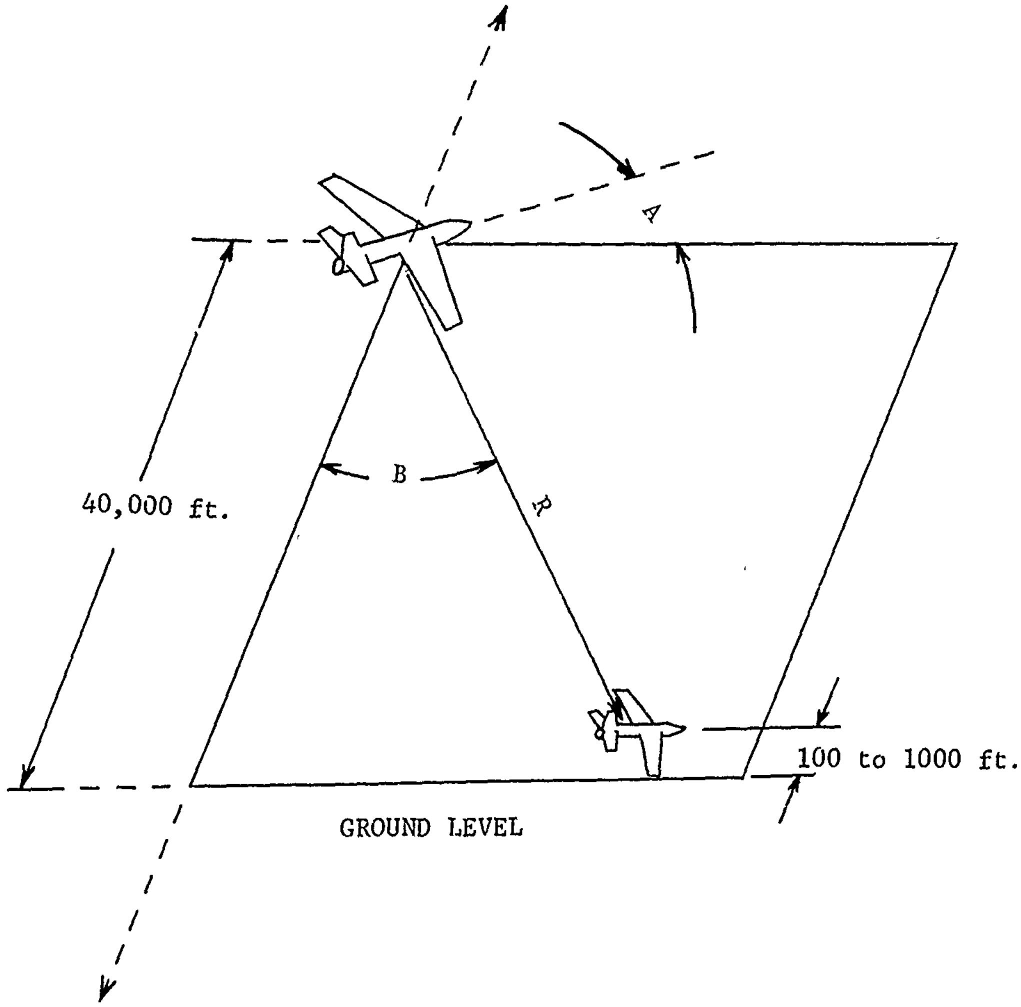


Figure 3. Configuration for Analysis of Airborne Tracker Errors.

assumed that correction can be made using the NBS atmospheric model to reduce the error to 10% of the uncorrected value [24] so that,

$$\sigma_{B2} \leq (1.43)(315)(0.1)\ \mu\text{rad}$$

$$\sigma_{B2} \stackrel{<}{=} 0.045 \text{ milliradian}$$

There is no corresponding bias error in the angle, A. There are, however, additional errors in both angles, A and B, caused by local fluctuations of refractivity, which can be estimated to be [8]:

$$\sigma_{A3} \cong 0.037 \text{ milliradian}$$

$$\sigma_{B3} \cong 0.037 \text{ milliradian}$$

A fourth error is attributable to the resolution of the radar beam and to equipment errors. It is some fraction of the 3-dB width of the beam. To estimate this error, consider [24] the AN/FPS-16, a ground based instrumentation radar which operates at 5.4 to 5.9 GHz. It has a 12-foot diameter reflector, a monopulse feed, and a Σ beam width of 1.1 degrees, or 19.2 milliradians. Its accuracy is characterized as nominally 0.1 mrad RMS bias and 0.1 mrad RMS noise, for a total of 0.14 mrad RMS, which is equivalent to 0.007 times the 3-dB beam width. It would be difficult to achieve the resolution and accuracy of the FPS-16 in an airborne radar even if the frequency is increased and the antenna scaled to fit in a 3-foot pod. The radome, the shorter wavelength, the necessarily lighter gimbal mounting, and the stringent requirements on servo drives would all add to both the bias and the noise errors in an airborne, small radar antenna.

The value of the transponder which would be mounted in the low-flying target can be seen from the following expression [24] for the error of a monopulse tracking radar:

$$\sigma_\theta = \frac{\theta}{k_m \sqrt{2nS/N}}$$

where

σ_θ = error in estimate of pointing angle

θ = 3-dB beam width of the beam

k_m = 1.63

= normalized monopulse slope

n = number of pulses integrated to estimate the pointing angle

S/N = signal-to-noise ratio of received signal

The upper limit on the pulse rate is range dependent, to avoid ambiguity, and would in our case be less than 4000 pps. At 600 mph the target would travel about 0.88 feet during four pulse intervals. For n = 4, and S/N = 20 dB,

$$\sigma_\theta = \frac{\theta}{1.63\sqrt{(8)(100)}}$$

$$= 0.022\ (\theta)$$

A reasonable design goal would appear to be that the errors in an airborne, mechanically pointed, millimeter radar would be about three times the FPS-16 errors, or 0.02 times the 3-dB radar beam width. This agrees with other estimates of realizable radar system accuracies of approximately one-fiftieth of the 3-dB beam width [38].

The 3-dB beam width of a monopulse radar, assuming $(\text{cosine})^2$ illumination, is [24]:

$$\theta = \frac{1.44\lambda}{w} \text{ radians}$$

where λ = radar wavelength, meters

w = aperture width, meters.

The tracking error, based on an estimate of one-fiftieth of the beam width, is:

$$\sigma_{A4} = \sigma_{B4}$$

$$= \frac{28.8\lambda}{w} \text{ milliradian}$$

If the antenna were a phased array, σ_{B4} would be increased by a foreshortening of the effective width of the antenna as the angle, B, increases:

$$\sigma_{B4} \text{ (phased array)} = \frac{28.8\lambda}{w \cos B}$$

Since it is not known whether a phased array antenna is feasible at 70 GHz and 95 GHz, only mechanically pointed antennas will be considered. If a phased array antenna is employed, the accuracy would be degraded by beam-spreading as the beam is deflected. If the antenna is a steered reflector, there is no beam spreading effect.

The total error for angle A is:

$$\sigma_{AT} = \left[\left(\sigma_{A1}\right)^2 + \left(\sigma_{A3}\right)^2 + \left(\sigma_{A4}\right)^2\right]^{1/2},$$

and the total error for angle B is:

$$\left(\sigma_{BT}\right) = \left[\left(\sigma_{B1}\right)^2 + \left(\sigma_{B2}\right)^2 + \left(\sigma_{B3}\right)^2 + \left(\sigma_{B4}\right)^2\right]^{1/2}$$

The range measurement error for an airborne radar would include all of the error sources of the range measurement in CIRIS, except that the errors in detecting the leading edge of the radar transponder pulse would be substituted for the multipath errors of the CW/DME system used in CIRIS.

A conservative estimate [9] of the error in a pulse leading edge ranging system is 5.7 feet, for a range of 120,000 feet. For a range of 100,000 feet, two distance-related error sources would be reduced, and the error in ranging to the lesser distance is estimated to be 5.6 feet. (A 1964 analysis of bias and noise static errors [25] estimated 4.5 feet RMS error can be achieved with an AN/FPS-16 radar.)

For small angles, the position errors at ranges less than 10^5 feet can be approximated by:

$$\Delta_A = R\sigma_{AT} \text{ feet}$$

$$\Delta_B = R\sigma_{BT} \text{ feet}$$

$$\Delta_R \leq 5.6 \text{ feet}$$

where,

R = range to target, feet

= H/cos B

σ_{AT}, σ_{BT} = angular errors, radians

Δ_R is based on a conservative estimate of range error of a pulse leading edge system [9]

H = height of airborne platform above target

B = LOS angle, measured from downward, vertical

It was estimated at the beginning of this analysis that the error budget for each axis of the target position is 8.14 feet, so the design criterion for the horizontal error due to angle A is

$$\Delta_A \leq 8.14 \text{ feet}$$

The horizontal error due to angle F for a one-meter antenna can be expressed as:

$$\begin{aligned}
\left(\Delta_A\right)^2 &= (R)^2\left(\sigma_{AT}\right)^2 \\
&= R^2\left[\left(\sigma_{A1}\right)^2 + \left(\sigma_{A3}\right)^2 + \left(\sigma_{A4}\right)^2\right] \\
&= \frac{(40,000)^2}{(\cos B)^2}\left[(0.078)^2 + (0.037)^2 + (29\lambda)^2\right] \times 10^{-6} \\
&= \frac{1600}{(\cos B)^2}\left[0.0061 + 0.0014 + (29\lambda)^2\right]
\end{aligned}$$

The minimum error, at a given wavelength, occurs with B = 0°, cos B = 1. The criterion for minimum horizontal error is:

$$\left(\Delta_A\right)^2_{min} \leq (8.14)^2 = 66.3$$

therefore,

$$(29\lambda)^2 \leq \frac{66.3}{1600} - 0.0075$$

and

$$(29\lambda)^2 \leq (0.184)^2$$

$$\lambda \leq \frac{0.184}{29}$$

$$\leq 0.64 \times 10^{-2}$$

The minimum frequency to meet this error criterion is:

$$f \geq \frac{3 \times 10^8}{0.64 \times 10^{-2}}$$

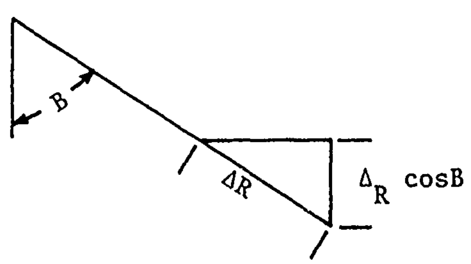
$$\geq 4.7 \times 10^{10}$$

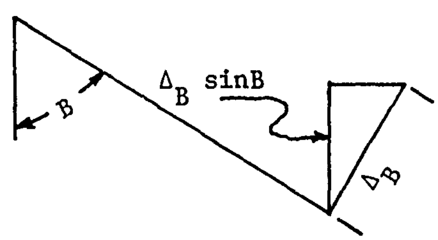
$$\geq 47 \text{ GHz}$$

Both 70 GHz and 95 GHz radars and a laser radar will be examined.

The vertical measurement of position is the most critical measurement for low-flying, terrain avoidance vehicles; therefore, the vertical error, Δ_Z, is the most critical error. It can be seen that the vertical components of Δ_R and Δ_B, in the vertical plane that contains the line-of-sight, are:

$$\Delta_{R(Z\ component)} = \Delta_R \cos B \qquad \Delta_{B(Z\ component)} = \Delta_B \sin B$$





Because it is orthogonal to Z, Δ_A has no effect on Δ_Z. The criterion for vertical error can be written,

$$\Delta_Z = \sqrt{(\Delta_R \cos B)^2 + (\Delta_B \sin B)^2} \leq 8.14 \text{ feet}$$

or,

$$(\Delta_Z)^2 = (5.6 \cos B)^2 + (R \sin B)^2 \left[(\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2\right] \leq 66.3$$

Noting that $R = \frac{H}{\cos B}$, where H = height of tracking aircraft above target, substitution may be made that:

$$R \sin B = H \tan B$$

and

$$(\Delta_Z)^2 = (5.6 \cos B)^2 + (H \tan B)^2 \left[(\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2\right] \leq 66.3$$

This criterion will be tested for H = 40,000 feet, first letting the operating frequency be 95 GHz, then 70 GHz.

The horizontal error in the vertical plane which contains the line-of-sight is orthogonal to Δ_A, and is:

$$(\Delta_{h,r}) = \sqrt{(\Delta_R \sin B)^2 + (\Delta_B \cos B)^2}$$

$$= \sqrt{(\Delta_R \sin B)^2 + \left[(R \cos B)(\sigma_{BT})\right]^2}$$

and, letting H = R cosB = 40,000 feet

$$(\Delta_{h,r})^2 = (5.6 \sin B)^2 + (40{,}000)^2 \left[(0.078)^2 + (0.045)^2 + (0.037)^2 + (\sigma_{B4})^2\right] \times 10^{-6}$$

$$= 31.4 \sin^2 B + 1600 \left[0.0095 + (\sigma_{B4})^2\right]$$

Since $(\Delta_{h,r})^2 \leq 66.3$, the criterion for this horizontal error is:

$$31.4 \sin^2 B + 15.2 + (1600)\ (\sigma_{B4})^2 \leq 66.3$$

$$31.4 \sin^2 B + 1600\ (\sigma_{B4})^2 \leq 51.1$$

$$\sin^2 B + 51(\sigma_{B4})^2 \leq 1.63$$

Noting that $\sin^2 B$ increases as B increases from zero, it is clearly sufficient to use this criterion along with the vertical error criterion to establish a maximum value for the deflection angle, B.

95 GHz Radar Design Parameters

For 55° deflection and f = 95 GHz, the values of the error components of the angle B are:

$$\sigma_{B1} = 0.078 \text{ milliradian}$$

$$\sigma_{B2} \leq 0.045 \text{ milliradian}$$

$$\sigma_{B3} = 0.037 \text{ milliradian}$$

and

$$\sigma_{B4} = \frac{(0.0288)(3)(10^8)}{(1)(95)(10^9)}$$

$$= 0.091 \text{ milliradian}$$

Testing the error criterion, with B = 55°, f = 95 GHz, and H = 40,000 feet,

$$(\Delta_Z)^2 = \left[(5.6)(0.574)\right]^2 + \left[(40{,}000)(1.428)\right]^2 \left[(0.078)^2 + (0.045)^2 + (0.037)^2 + (0.091)^2\right] \times 10^{-6}$$

$$= 10.3 + (1600)(2.04) \left[0.0061 + 0.0020 + 0.0014 + 0.0083\right]$$

$$= 10.3 + 58.1$$

$$= 68.4 \nleq 66.3$$

However, with a mechanically steered antenna holding the target at boresight, the vertical error criterion can be shown to be met if the deflection angle, B, is 54 degrees or less. When B = 54°,

$$(\Delta_Z)^2 = \left[(5.6)(0.588)\right]^2 + (1600)(1.376)^2 \left[0.0178\right]$$
$$= 10.8 + 53.9$$
$$= 64.7 < 66.3$$

Testing the criterion for horizontal error, when B = 55° and f = 95 GHz:

$$\sin^2 B + 51(\sigma_{B4})^2 = (0.819)^2 + 51(0.091)^2$$
$$= 0.671 + (0.51(0.83)$$
$$= 1.09 < 1.69$$

It is seen that at an operating frequency of 95 GHz, if a 1 meter antenna is used and the altitude of the CIRIS platform is 40,000 above the low-flying target, and if the line-of-sight deflection angle remains less than 54 degrees, the vertical target position error will not exceed 10 feet, RMS. Range from the airborne platform to the low-flying target would be 68,000 feet maximum.

70 GHz Radar Design Parameters

If the operating frequency of the airborne radar is 70 GHz, σ_{B4} will be increased:

$$\sigma_{B4} = (0.091)\left(\frac{90}{70}\right)$$
$$= 0.118 \text{ millirad.}$$

Testing the vertical error criterion for 70 GHz operating frequency, and B = 50 degrees,

$$(\Delta_Z)^2 = \left[(5.6)(0.643)\right]^2 + (1600)(1.19)^2 \left[0.0061 + 0.0020 + 0.0014 + (0.118)^2\right]$$

$$= 13.0 + (2272)(0.0234)$$

$$= 13.0 + 53.2$$

$$= 66.2 < 66.3$$

The horizontal criterion for f = 70 GHz and B = 50 degrees can also be tested:

$$\sin^2 B + 51(\sigma_{B4})^2 = 0.587 + 0.710$$

$$= 1.30 < 1.64$$

Laser Radar

A conceptual airborne laser radar boresighted with a conventional radar is suggested by a recent article which describes such a combination in a ground based installation [39] and by a description of a laser spacecraft communication system [40]. The conventional radar would be used for target acquisition, and final target lock-on and dish steering would be performed by the laser. The design parameters for the laser radar can be estimated.

The altitude error estimated for the CIRIS type reference platform was:

$$\sigma_{B1} = 0.078 \text{ milliradian}$$

The errors due to atmospheric refraction can be extrapolated by noting how the atmospheric model for radio frequencies differs from the model for optical frequencies [34]. For radio frequencies, the atmospheric index of refraction is:

$$n_R = 1 + \left[\frac{77.6P}{T} + \frac{3.73 \times 10^5 e}{T^2}\right] \times 10^{-6}$$

where

n_R = index of refraction at radio frequencies

P = atmospheric pressure, millibars

T = temperature, degrees Kelvin

e = partial pressure of water vapor, millibars

For optical and infrared frequencies the index of refraction is:

$$n_0 = 1 + \left[\frac{77.6P}{T} + \frac{0.584P}{T\lambda^2}\right] \times 10^{-6}$$

where

λ = wavelength, microns

n_0 = index of refraction at optical frequencies

Noting that the expressions differ in the last term only, let

$$\frac{0.584P}{T\lambda^2} = \frac{3.73 \times 10^5 e}{T^2}$$

and solve for λ to obtain an expression that represents equivalence of water vapor and wavelength effects on refractivity:

$$\lambda = 1.25 \times 10^{-3} \sqrt{\frac{PT}{e}}$$

Letting P, T, and e have the following ranges:

$$950 < P < 1050 \text{ mb}$$
$$1 < e < 30 \text{ mb}$$
$$274 < T < 318 \text{ deg K}$$

the corresponding range in λ would then be:

$$0.12 < \lambda < 0.72 \text{ microns.}$$

This represents the range of optical wavelengths which contribute to atmospheric refractivity the same error as contributed by water vapor from 1 to 30 mb partial pressure. Wavelengths longer than 0.72 microns will have less effect on refractivity than 0.1 percent water vapor. The radio frequency model which was used to estimate position errors of the CIRIS type airborne reference assumed standard atmospheric conditions of temperature (288°K), pressure (1013 mb), and about 1.0 percent water vapor, or a partial pressure of 10.2 millibars (relative humidity 60 percent). The refractive index for <u>radio</u> frequencies under these conditions is:

$$n_R = 1 + 77.6 \left[\frac{1013}{288} + \frac{(4807)(10.2)}{(288)^2}\right] \times 10^{-6}$$

$$= 1 + 77.6\ (3.518 + 0.5911) \times 10^{-6}$$

If the water vapor content were reduced by a factor of ten, to one millibar, the refractive index would be

$$n_{R_{(dry)}} = 1 + 77.6\ (3.518 \times 0.0591) \times 10^{-6}$$

and the error term for conventional radar would in this lower R.H. case be reduced by 12.9%. If the laser radar operates at 1.06 micron, the error for refraction of the atmosphere can be extrapolated from the estimates of refractivity error for radar:

$$\sigma_{B2} \leq (0.045)(0.875)$$

$$= 0.039$$

$$\sigma_{B3} < (0.037)(0.875)$$

$$= 0.032$$

The estimate of the beam width error in the case of radar included pointing resolution error, equipment error, multipath effects, etc. It would be possible, though not desirable, to make the laser beam extremely sharp. Instead, it should be made broad enough to maintain illumination of a retroreflector mounted on the target. The beam width requirement would be defined by the optics of the system, the detector, and the system dynamics. The basic RMS pointing error, in a diffraction limited system, has been expressed as [40]:

$$\sigma_\theta = \frac{1.22\lambda}{D\ (S/N)} \text{ radian}$$

where

λ = wavelength, meters, = 1.06×10^{-6}

D = aperture of optics, meters

S/N = signal-to-noise ratio, assumed to be greater than 10.

Thus,

$$\sigma_\theta < \frac{1.3 \times 10^{-4}}{D} \text{ millirad}$$

and if D > 10 cm,

$$\sigma_\theta < 0.001 \text{ milliradian}$$

As we have noted, however, the error that is equivalent to the radar beam width error would include equipment errors. The "absolute accuracy" specification for a commercially available ground based laser radar, PATS (Precision Automated Tracking System), is:

$$\sigma_{B4} = 0.1 \text{ milliradian.}$$

The retroreflector used by PATS is 3 inches in diameter, which subtends an arc at a range of 50,000 feet of only 0.005 milliradian. Improvement of σ_{B4} to 0.02 milliradian instead of 0.1 milliradian might be feasible, but system dynamics (control) errors would probably be limiting.

The range error for laser radar should be considerably less than the 5.6 feet assumed for radar, because there is no delay uncertainty associated with the transponder, and because the range/timing pulse can be of the order of one to ten nanoseconds. The primary sources of range error are the residual error from the corrected atmospheric index of refraction, plus counter logic (start/stop) uncertainty. These sources, in PATS, produce an "absolute" range error specification of 2 feet at 65,000 feet.

The vertical error criterion of 8.14 feet can now be tested, letting B = 60°, and using the following error values:

$$\sigma_{B1} = 0.078 \text{ milliradian}$$

$$\sigma_{B2} = 0.039 \text{ milliradian}$$

$$\sigma_{B3} = 0.032 \text{ milliradian}$$

$$\sigma_{B4} = 0.1 \quad \text{milliradian}$$

$$\sigma_Z = \sqrt{(\Delta_R \cos B)^2 + (\Delta_B \sin B)^2} \leq 8.14$$

$$= \sqrt{(\Delta_R \cos B)^2 + (H \tan B \; \sigma_{BT})^2}$$

$$(\Delta_Z)^2 = \left[(2)(0.5)\right]^2 + (1600)(3) \left[(.078)^2 + (0.039)^2 + (0.032)^2 + (0.1)^2\right]$$

$$= 1 + 89.4 \nleq 66.3$$

The test fails for B = 60°, but for B = 55 degrees:

$$(\Delta_Z)^2 = \left[(2) \; (.574)\right]^2 + (3263)(0.0186)$$

$$= 1.3 + 60.7$$

$$= 62.0 < 66.3$$

With a laser radar, deflection from vertical downward direction would have to be less than 55 degrees to keep the vertical error in the target position less than 10 feet RMS.

System Constraints

One consequence of the restrictions on the angle B is that the ratio of the speed of the target vehicle to the speed of the tracking aircraft is also restricted. Assume, for example, the tracking aircraft is 40,000 feet above ground level, the tracking radar is 70 GHz, and the test flight begins with the low-flying vehicle 48,000 feet (horizontal distance) behind the tracking aircraft (B = 50°). The target would pass under the tracking aircraft at midcourse and at the end of a 600,000-foot run would be 48,000 feet in the lead, only if the target is 16 percent faster than the tracker. If the target is too fast it will pull too far ahead, the angle B would exceed 50°, and the error in the estimate of target altitude would become excessive.

Summary

It has been established that if the system errors are as estimated, a 70 GHz radar, a 95 GHz radar, or a laser radar would permit measurements of position of a low-flying target from a CIRIS type airborne platform, with errors in each of three orthogonal axes (two horizontal and one vertical) less than 10 feet, RMS. Table I summarizes the system constraints on line-of-sight angles.

Some of the sources of error, such as the servos which point the radar reflector and the delay uncertainty of the transponder mounted in the low-flying target, have been assumed to be included in the estimates of σ_{B4} and Δ_R. If the error estimates are too small, the angular deflection, B, would have to be reduced, the frequency of the operation raised, or the operating altitude lowered. These parameters, together with their effects on the necessary ground based transponder configuration, constitute trade-off components.

A system constraint that has been established is that the speed of the low-flying target cannot greatly exceed the speed of the tracking aircraft because the angle of the line-of-sight from the platform to the target must be held within about 50 degrees from the downward vertical direction. With this constraint a 600 mph aircraft could track a target only if the target does not exceed 715 mph.

TABLE I

DESIGN CONSTRAINTS FOR AIRBORNE RADARS THAT PERMIT MEASURING POSITION OF LOW-FLYING TARGE WITH ACCURACY OF 17.3 FEET, RMS, VECTOR

Frequency/ Wavelength	Antenna	Maximum LOS Angle Degrees	Maximum Range	Maximum Ground Projection of Range	Ratio of Target Speed to CIRIS Platform Speed*
95 GHz	Reflector	54	1.70H	1.38H	1.22:1
70 GHz	Reflector	50	1.55H	1.19H	1.19:1
1.06 micron	Laser	55	1.74H	1.43H	1.24:1

*Assumes H = 40,000 feet, total run = 600,000 feet.

The most influential vertical error component is a scale factor multiplier, H tanB, where H is the height of the tracking aircraft above the target, and B is the angle of the line-of-sight measured from downward vertical. This is a range related error, inasmuch as H tanB is the projection onto the ground of the range from the tracking aircraft to the low-flying target. If the maximum deflection of B is 55°, and the tracking aircraft is at 40,000 feet AGL, maximum range to target is about 57,000 feet.

Appendix E describes state-of-the-art 95 GHz and 70 GHz prototype radars that have been fabricated and evaluated by Georgia Tech. The state-of-the-art in laser ranging and tracking is described in the literature [39,40].

R and D Program

To develop an airborne radar tracking system, for closing the link from aircraft to target in the system of Figure 1, will require a major R & D effort. A significant part of that effort would be devoted to the development of a tracking antenna and radome, for mounting in an airborne pod. The pointing mechanism would have to be capable of pointing the antenna with an accuracy of about 10 $\widehat{\text{sec}}$ in each of two orthogonal axes, and reading out the angles with comparable accuracy. This is approximately the performance level of the AN/FPS-16 radar.

The advantages of a phased array antenna for tracking low-flying vehicles are enumerated in Appendix B, and those advantages would probably outweigh the drawback of beam spreading and loss of resolution as the beam is deflected from normal to the array plane. However, there are no phased array millimeter wavelength antennas known to the authors. The development of such an antenna would be a higher risk and would cost more than the development of a steered dish antenna. Indeed, it would first have to be determined that phased array antenna elements are available or feasible at 70 GHz or 95 GHz.

The development of an airborne laser tracking radar to be boresighted with a Ku band acquisition radar would be an R and D effort comparable to developing an airborne millimeter radar.

4. SYSTEMS BASED ON AVAILABLE EQUIPMENT

Three systems have been conceived that could partially meet the WSMR requirements using available equipment. One of these is a totally ground based system using the Sylvania laser tracking radar (PATS), which is commercially available. The second conceptual system would be like that of Figure 1, with a Ku band radar in the tracking aircraft, or some means of maintaining the tracker as near as possible directly above the low-flying vehicle. The third system would be a multilateration ranging system; target position would be computed from radio range signals between the target and at least three ground stations, between the target and the airborne platform, and between the airborne platform and the ground stations.

The ground-based laser radar network could meet the WSMR requirements for accuracy, at a cost that would be high but perhaps not beyond reason. The station keeping Ku band radar would permit ranging measurements of the altitude of the low-flying vehicle within the WSMR accuracy requirements, but horizontal location errors would be on the order of 40 feet RMS. An analysis of errors in the "pyramidal" multilateration system indicates acceptable accuracy can be obtained, with "good" atmospheric conditions.

Ground Based Laser Tracking System

The Sylvania laser radar (PATS) specifies a range accuracy of 2 feet and an angular resolution of 0.1 milliradian about two axes out to 65,000 feet of range. The tracking rate is about 30 degrees per second.

A PATS laser radar has been observed tracking a helicopter at about 36,000 feet range, alternately against sky and desert background. The laser radar was sometimes depressed below horizontal. Acquisition was accomplished with the aid of a video camera which was boresighted with the laser. The operator first nominally centered the target on a TV screen with joystick control, then he activated laser lock-on.

The jitter of the target as seen on the TV display appeared to be about 5 feet. This corresponds to a pointing angle error of 1/7 milliradian.

A video recording made during earlier tests showed PATS tracking a mortar shell (fitted with a retroreflector) at the same distance of 36,000 feet.

A system consisting of nine or ten such laser radars positioned every 10 miles along the test flight path, and set back about 4 miles, would meet the WSMR requirements for location accuracy.

The cost of one tracker is about $600,000, but the unit cost of several such systems could be considerably less, enough so to consider such a system.

Each laser radar station would locate the target vehicle with respect to its own position. Range, azimuth, and elevation are measured. The reduction of these data to location of the target on a master coordinate system is a modest computation which must be made at each station in order to supply the next station orders for acquisition.

A minimum layout of 9 or 10 trackers, while not covering the whole of WSMR, would provide a test corridor which could include a number of alternative paths.

As supplied to Yuma, the PATS angular tracking rate (500mr/sec) is adequate for targets to above Mach two if the crossing range is kept greater than about 5000 feet.

At the laser wavelength of 1.06 microns there is no multipath error, because surface roughness scatters the signal instead of reflecting it, and the retroreflector is a very efficient transponder. Range scale error should be only about a foot at 10^5 feet, after bias error due to atmospheric refraction has been corrected by using a model for the atmosphere. Angular error associated with refraction and scintillation will limit the system when the air is unstable over the line-of-sight, and it may be that the times of worst atmospheric conditions must be avoided. (This would be true for conventional radar systems, too.) About ninety percent of the bias error can be removed by refractivity correction, and straightforward smoothing of the target trajectory will reduce the effects of residual fluctuations due to local atmospheric turbulence.

It may be desirable to record both the angular encoder outputs and the error signal, rather than have the trajectory smoothed by the response of the servo systems and the smoothing filters that are used in the present installation. Subsequent smoothing of the recorded data could be more effective than real time smoothing. The tape recording fc mat of PATS is compatible with computer processing.

If modest improvements can be made to reduce the pointing angle errors below the 0.1 milliradian that the manufacturer claims, the PATS laser units could replace theodolites as basic range instruments. Data turn-around time could be greatly reduced.

Overhead Tracking, Ku Band

The measurement of altitude of low-flying, terrain avoidance vehicles is usually more critical than the determination of horizontal position coordinates. If the accuracy requirement for horizontal position determination is relaxed, the frequency of the radar for the system described in Chapter 3 could be as low as 17 GHz. The antenna could also be smaller than one meter, say 0.5 meter. It could also be rigidly mounted and pointed nominally downward, with a display for the pilot which would enable him to maintain the aircraft very nearly over the low-flying vehicle. When sighting directly down, the vertical error in the ranging signal could be estimated as 5.6 feet, using the same assumptions as in Chapter 3. The error normal to the line-of-sight would be:

$$\Delta_B = H\sigma_{BT}$$

where

$$\sigma_{BT} = \sqrt{(\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2} \quad \text{radians}$$

and

$$\sigma_{B1} = 0.078 \text{ milliradian}$$

$$\sigma_{B2} = 0.045 \text{ milliradian}$$

$$\sigma_{B3} = 0.037 \text{ milliradian}$$

$$\sigma_{B4} = \frac{(1.44)(3)(10^8)}{(50)(0.5)(17.5)(10^9)}$$

$$\cong 1 \text{ milliradian}$$

Therefore, if H = 40,000 feet

$$\Delta_B = 40\sqrt{(0.078)^2 + (0.045)^2 + (0.037)^2 + (1.0)^2}$$

$$= 40\sqrt{(0.0061 + 0.0020 + 0.0014 + 1.0)}$$

$$\cong 40 \text{ feet}$$

As the beam deflects from directly downward, for small angle θ the vertical error criterion is:

$$(\Delta_Z)^2 = (5.6 \cos \theta)^2 + (40 \sin \theta)^2$$

$$= \cos^2\theta \left[(5.6)^2 + (40 \tan \theta)^2\right]$$

$$\leq 66.3$$

But for $\theta \leq 10°$, $\cos^2\theta \cong 1.0$, and

$$\tan \theta \leq 0.148$$

$$\theta \leq 8.4°$$

By using a sufficient number of ground sites, placed to minimize vertical error in determining the position of the tracking aircraft, the error in estimating the altitude of the low-flying vehicle can be held to less than 10 feet RMS. Eighteen ground sites placed at the corners of squares as shown in Figure 2 would seem to be sufficient for a flight-path corridor.

The following section examines the possibility of utilizing only ranging measurements, from ground sites to the target and to the over-flying aircraft, and also from the aircraft to the target.

Multilateration with Two Vehicles

If one attempts to determine the position of a low-flying vehicle from ground-based radio ranging signals, five categories of error sources affect the accuracy. One error category is associated with the modulation type. If FSK FM is used, multipath causes phase error in the demodulated signal, and hence causes error in range estimates. If pulse modulation is used, the pulse becomes distorted in transmission and detection error results. The second category of error is associated with equipment noise and delays. A third source of error is related to the inability to completely compensate for the atmospheric index of refraction. This error is directly proportional to range; hence, it is a scale factor error. The fourth error category is due to fluctuations of the index of refraction along the path of propagation. It is related to measurement time interval. The effect is negligible for sampling rates faster than one per minute, and for distances less than 100 miles. The fifth error category is survey error, which produces uncertainty in the location of ground stations.

The technique of Figure 1 would use multilateration to estimate the position of an over-flying reference aircraft, and would estimate the position of the target relative to the reference aircraft by radar measurements of range and pointing angles. Analysis of this system, previously discussed, has shown that present state-of-the-art airborne radar equipment cannot meet the error specifications desired.

Another approach that is attractive would use multilateration from ground based stations to locate both the over-flying reference aircraft and the low-flying target, with only the horizontal coordinate estimates of the target being retained. The target altitude could then be calculated from range measurements made from the chase ship to the target. Such a system would avoid the large uncertainties associated with angular measurements made with available airborne radar. The following analysis considers such a system and uses a physical approach to develop estimates of the errors.

It will be shown that a radio ranging, multilateration approach can yield measurements of position of the low-flying target with 10-foot RMS accuracy in any axis. The aircraft must stay within about 20,000 feet, horizontally, of an overhead position above the target to maintain the desired altitude measurement accuracy.

The over-flying aircraft will be located by lateration from only three stations in the example presented. Redundant measurements from additional ground stations could improve the position estimates.

The test vehicle will be located by lateration from ground stations but only its horizontal position will be further utilized, the altitude being too poor a result to retain.

Thus, six range signals locate the aircraft in space and the test vehicle horizontally. A seventh range signal from the aircraft to the test vehicle permits the altitude of the test craft to be determined.

The propagation of each range measurement uncertainty into this altitude determination will be developed. An estimate of precision of the horizontal location will also be made.

In actual use such a system should perform better than the analysis indicates because redundant data will sometimes be taken from more than three ground stations at a time. Also, after the altitude of the test craft is determined as described, the horizontal location would be recalculated, again using the original information but this time with a redundant range to a station. Such iteration would improve the results.

Propagation of Uncertainties

When the effect of an uncertainty in range from a ground station to an airborne vehicle affects a derived quantity, one needs to know the sensitivity of the latter to the former. In our case the derived quantity will be a position coordinate of the vehicle such as altitude. If a mathematical expression is available relating altitude to range from the three stations then the sensitivity is represented by $(\partial Z/\partial r_k)$, where Z is the altitude and r_k is the range from the station in question.

In order to avoid developing a complete expression $Z = f(r_1, r_2, r_3)$, and to keep a strong physical meaning attached to each step, a computation of $\partial Z/\partial r$ will be made directly. Figure 4 shows any three ground stations A, B and C. We seek $\partial Z/\partial r = (\partial Z/\partial r_A)_{r_B r_C}$. Physically, if r_B and r_C are maintained fixed, then the position of the intersection of r_A, r_B, and r_C at D must lie on a circle in a vertical plane that is centered on the line joining B and C.

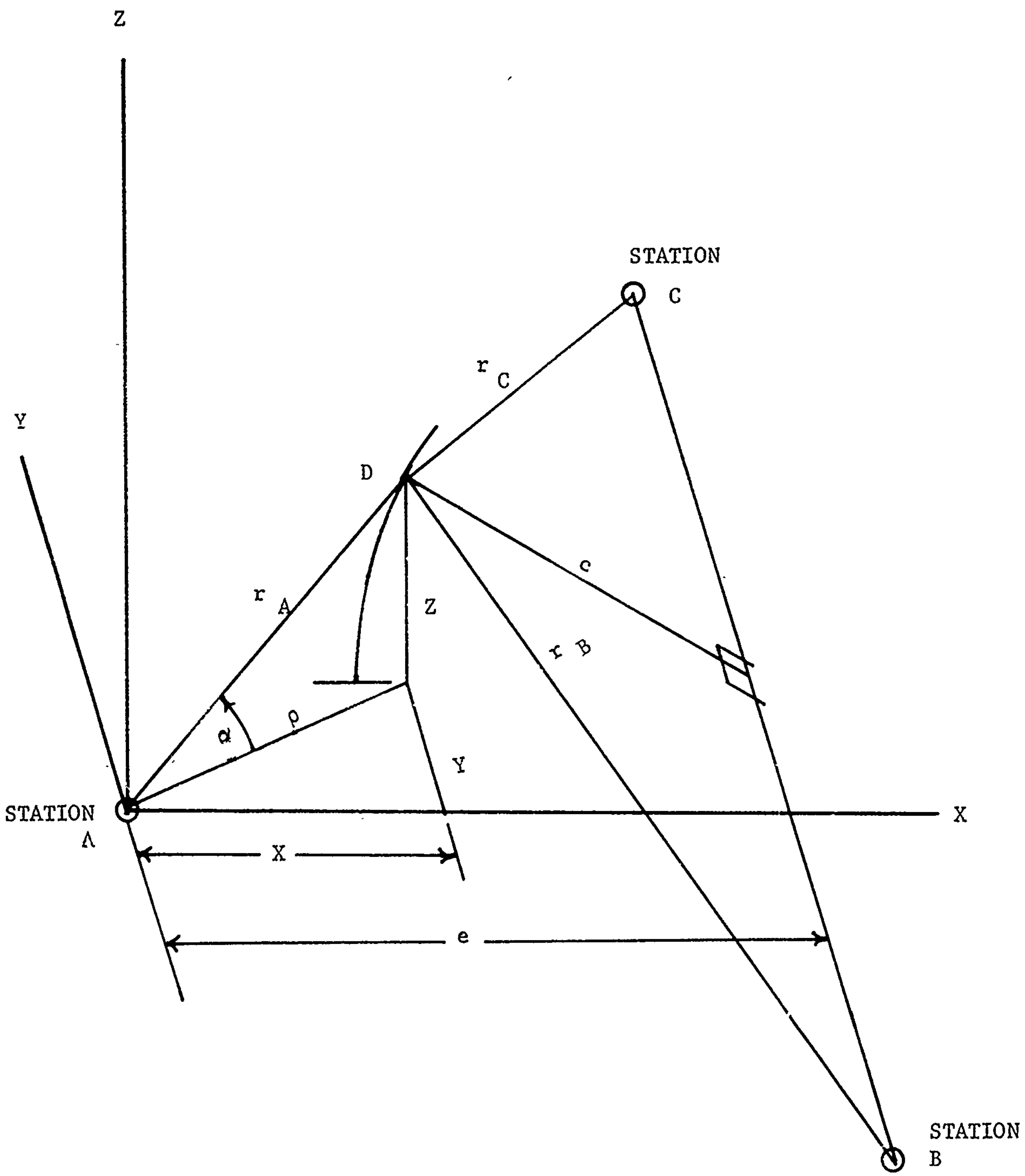


Figure 4. Configuration for Analysis of Multilateration System Errors.

The radius is the line of fixed length, c; c is determined by the sides of the triangle DBC, all three sides of which are of fixed length, and by the requirement that the line pass through D and be perpendicular to BC. Considering B and C to be on level ground for this idealization, BC is a horizontal line and the plane in which c is restrained to turn is a vertical plane.

In order to write analytic expressions a coordinate system has to be selected. Choose Z as vertical, the origin at station A, the direction Y parallel to BC. The plane containing line c is a plane described by Y = const. D is constrained to a circle on this plane as shown in Figure 4. The equation of this circle is,

$$c^2 = (e - X)^2 + Z^2 \tag{1}$$

where e is the X-coordinate of stations B and C.

When $r_A = r$ is determined, D is fixed, locating our vehicle. Determination of r fixes D on a sphere of radius r centered at 0, (point A) given by

$$r^2 = X^2 + Y^2 + Z^2 \tag{2}$$

A condition satisfying (1) and (2) simultaneously is a relation associated with the position of D, the intersection of the sphere with the circle.

There are many manipulations of (1) and (2) that will give the following results. In our analysis Y, c, e are considered fixed quantities, while r is allowed to vary. It is clear that any variation is r is accomplished by variation in both X and Z, so uncertainty in Z arising out of uncertainty in r is correlated with uncertainty in X.

The mutual dependence of Z and X can be derived by differentiating (1),

$$\begin{aligned} 0 &= -2(e - X) + 2Z\frac{\partial Z}{\partial X} \;, \\ \frac{\partial Z}{\partial X} &= \frac{e - X}{Z} \end{aligned} \tag{3}$$

The dependence of X on r can be obtained by differentiating the expression obtained by substituting the value of $(X^2 + Z^2)$ from (2) into (1),

$$c^2 = e^2 - 2eX + r^2 - Y^2 \quad ,$$

$$0 = -2e \frac{\partial X}{\partial r} + 2\, r \quad , \tag{4}$$

$$\frac{\partial X}{\partial r} = \frac{r}{e}$$

Combining equations (4) and (3) gives

$$\frac{\partial Z}{\partial r} = \frac{e - X}{Z} \; \frac{r}{e} = \frac{e - X}{e} \; \frac{r}{Z} \quad ,$$

$$\frac{\partial Z}{\partial r} = \frac{1}{\sin \alpha} \; \frac{e - X}{e} \quad , \tag{5}$$

where α is the elevation angle of D from the ground station, as shown in Figure 4. Since all effects due to r lie in the plane Y = const, then

$$\frac{\partial Y}{\partial r} = 0 \tag{6}$$

All horizontal uncertainty associated with range measurement from a particular station lies in a direction perpendicular to the line joining the other two stations.

In computing the uncertainty of position of the airborne craft at D, there will be uncertainties in r, which must be summed up, such as the scale factor error due to uncertainty of the index of refraction over the transmission path, the errors due to measurement instrument noise and multipath, and the error due to instrument bias.

Another source of error is in the survey of the ground station itself. The effect of the survey error on the location of the origin in Figure 4, should the displacement be along an arc centered under D and perpendicular to ρ, will have no effect on the computation of D. The measured r will have the same value and the same uncertainties as if it were measured from the assumed origin, and the resulting computation of the location of D will not be affected.

Should the dislocation of the origin be along ρ then the r being measured is not the r from the assumed origin. Figure 5 is a sketch of the vertical plane containing both ρ and r. The displacement of the station along ρ increases the measured r by

$$\delta r = \delta \rho \quad \cos \alpha \tag{7}$$

The effect on the computations will be the same as mismeasuring r by the amount given by (7). Ordinarily, survey errors are uncorrelated with range measurements, so an uncorrelated uncertainty of the amount indicated in (7) must be added to other errors associated with r.

If there is vertical as well as horizontal survey uncertainty, its component parallel to the range vector will contribute to position computation just like the parallel component of horizontal survey error, (7). No vertical survey error will be included in this analysis.

The partial derivatives, and all the contributions to δr just discussed are not limited to the coordinate system of Figure 4; it is only necessary that their proper geometrical meanings be recognized. To get the full uncertainty in the altitude, Z, and the uncertainties in horizontal position, it will be necessary to account for the other two stations, whose range measurements are partially correlated, as will be seen. To do this, we select a single coordinate system, (x, y, z) for all three stations. For each station we compute the uncertainty in x_A. y_A, and z_A, the coordinates of the aircraft in this system, due to each of the stations.

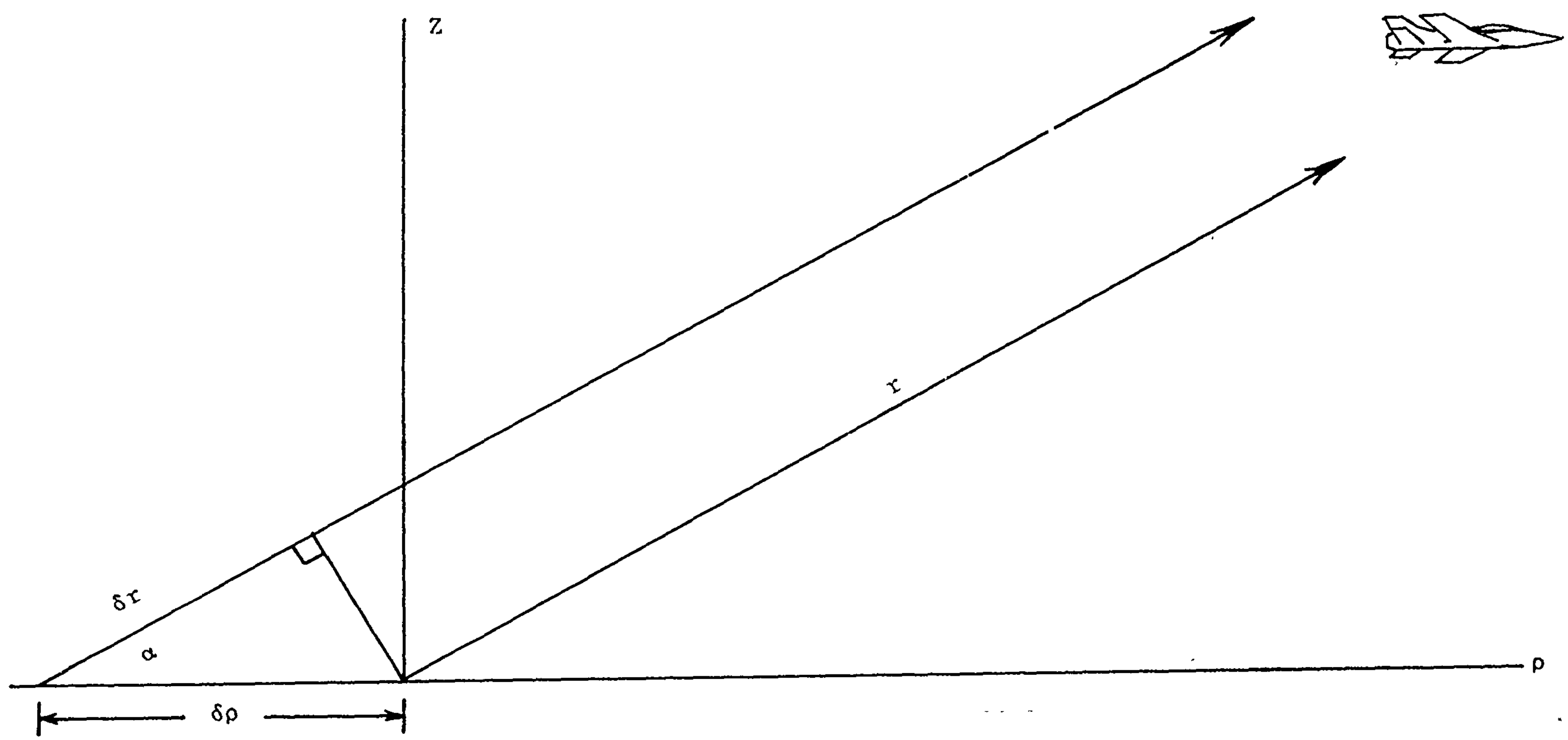


Figure 5. Effect of Displacement (Survey Error) of Ground Site.

Thus

$$\delta x_A = \frac{\partial x_A}{\partial r_1}\,\delta r_{A1} + \frac{\partial x_A}{\partial r_2}\,\delta r_{A2} + \frac{\partial x_A}{\partial r_3}\,\delta r_{A3} \qquad (8)$$

and likewise for δy_A and δz_A. The subscript, A, refers to the overflying aircraft. Thus, r_{A2} refers to the range to the aircraft from station 2. Here $\frac{\partial x_A}{\partial r_1}$ is the component of $\frac{\partial X}{\partial r}$, of (4), parallel to the x axis, etc. Use of expressions like (8) to compute the covariance elements of the aircraft position will be described below. As developed, $\frac{\partial x_A}{\partial r_1}$, etc., are differential expressions for displacements arising from any source.

Our present interest is to estimate the quality of a location of the test vehicle below the aircraft, rather then the location of the aircraft itself. The position of the test vehicle will first be determined as was that of the airplane, although the vertical uncertainty will be very large due to the low values of α. This altitude uncertainty will be so great that the vertical determination will be abandoned, and used no further in the computations; and only the horizontal position estimates will be retained. The uncertainties in the ranges from the three ground stations to the test vehicle will probably be larger than the errors in ranging to the aircraft because of the greater uncertainty due to multipath.

Altitude of the test vehicle will be determined from an additional radio ranging from the overflying aircraft. Its altitude will be given by

$$z_T = z_A - r_a \cos\beta \qquad (9)$$

where β is the angle between the vector from the aircraft to the test craft and the vertical. The range from the aircraft to the test vehicle is r_a and the aircraft altitude is z_A. We need (9) in the form where it depends upon the seven range measurements used to compute it. Let the horizontal displacement of the test vehicle relative to the aircraft be

$$h = \sqrt{\xi^2 + \eta^2} = \sqrt{(x_T - x_A)^2 + (y_T - y_A)^2} \qquad (10)$$

then

$$\beta = \sin^{-1} \frac{h}{r_a} \tag{11}$$

and (7) becomes

$$z_T = z_a - r_a \cos \left(\sin^{-1} \frac{\sqrt{\xi^2 + \eta^2}}{r_a} \right) \tag{12}$$

The partial derivatives of interest come out of (12):

$$\frac{\partial z_T}{\partial z_a} = 1 \tag{13a}$$

$$\frac{\partial z_T}{\partial r_a} = - \cos\beta - r_a \left(\sin \sin^{-1} \frac{\sqrt{\xi^2 + \eta^2}}{r_a} \right) \left(\frac{\sqrt{\xi^2 + \eta^2}}{r_a^2 \sqrt{1 - \frac{\xi^2 + \eta^2}{r_a^2}}} \right) \tag{13b}$$

$$= - \cos\beta - \frac{\sin^2\beta}{\cos\beta} = - \frac{1}{\cos\beta} \left(\cos^2\beta + \sin^2\beta\right) = \frac{-1}{\cos\beta}$$

$$\frac{\partial z_T}{\partial \xi} = r_a \sin\beta \frac{\xi}{\sqrt{1 - \frac{\xi^2 + \eta^2}{r_a^2}} \; r_a \sqrt{\xi^2 + \eta^2}} = \frac{\xi}{r_a \cos\beta} \tag{14}$$

Similarly,

$$\frac{\partial z_T}{\partial \eta} = \frac{\eta}{r_a \cos\beta} \tag{15}$$

An uncertainty in z_T can be expressed,

$$\delta z_T = \left(\frac{\partial z_T}{\partial z_a}\right) \delta z_a + \left(\frac{\partial z_T}{\partial r_a}\right) \delta r_a + \left(\frac{\partial z_T}{\partial \xi}\right) \delta \xi + \left(\frac{\partial z_T}{\partial \eta}\right) \delta \eta \tag{16}$$

$$\delta z_T = (\delta z_A) - \frac{1}{\cos\beta}\,\delta r_a + \frac{\xi}{r_a \cos\beta}\,\delta\xi + \frac{\eta}{r_a \cos\beta}\,\delta\eta \tag{16}$$

where (δz_a), $\delta\xi$, and $\delta\eta$ need to be expanded in terms of the six ranges measured from the three stations.

The Measurement Uncertainties

The range uncertainty from a ground station to the overflying aircraft can be due to serveral sources. For any one station

$$\begin{aligned}\delta r_A &= \delta r_{A,N} + \delta r_{A,B} + \delta r_{A,R} + \delta r_{A,S} \\ &= \delta r_{A,N} + \delta r_{A,B} + \delta r_{A,R} + \delta\rho \cos\alpha \end{aligned} \tag{17}$$

where subscript B refers to bias in the instrument zeros due to adjustment tolerances and drift, N to noise and multipath, S to station survey uncertainty, and R to scale factor error associated with uncertainty in the index of refraction over the path.

There will be a differential expression like (17) for each of the three ranges to the aircraft, and likewise for each of the three ranges to the test craft. The range, r_a, from the overflying aircraft to the test vehicle has a similar expression except there is no survey error term. Twenty-seven sources of uncertainty have been identified, four for each of six range measurements from the ground stations, and three for the range measurement between the two vehicles. It would be straightforward but unwieldy to expand the differential expressions for δz_T, δx_A, etc. in terms of these 27 uncertainties. Rather, we will collapse the 27 into the covariance elements of the seven range measurements and develop the uncertainty in vehicle location from the expressions already set down.

Correlations

The four sources of uncertainty identified in (17) are independent in any one range measurement, and therefore no correlation between them is to be expected. Also, no correlation is to be expected between sources of different type in different range measurements. However, certain correlations do exist between errors arising from the same type source, in the different range measurements, and these correlations are too strong to be ignored.

Noise in the querying device and in the transponder lead to timing errors and therefore to range errors. Different range measurements will normally be made at different times, so there will be no correlation of the noise of one range measurement with that of another.

Multipath consists of addition of direct path ranging signals with reflected signals. The reflected signals can pull tne time measurement either way, depending on their phase. This is true when complex coding such as PSK/FM is employed, and also when very short pulse signals are used. Multipath error depends in a complex way, but strongly, on the detailed geometry of the location of the interrogator and the transponder, and on the surrounding topographical feature. Correlation of multipath errors between range measurements over different paths is not expected.

Noise and multipath can be lumped together in the computations which follow because they constitute all of the sources considered that correlate with no other sources.

The three remaining sources, while not correlating with each other, are correlated between some paths. The most obvious is survey error of the stations. We envision range measurements to the overflying aircraft and to the test vehicle from each of the ground stations. The test vehicle will be at nearly the same azimuth from any one station, so there is strong correlation between $\delta r_{A,S}$ and $\delta r_{T,S}$ from any one station. Whatever the error is, the two will be proportional to one another and have the same sign, therefore a correlation coefficient of one can be used. (It is possible to account for the slightly different expected azimuth angles by using the average cosine of the azimuth difference, instead of using one; but this average cosine will typically exceed 0.95 in the system under consideration.)

Bias error in any range measurement is associated with alignment tolerances and subsequent circuit drifts which vary time delays in the transponder, the interrogating transmitter, and the receiver. Each range to be measured will share a transponder or interrogator with another range, and the bias in the common device will correlate between these ranges. The expected magnitude of bias in a transponder will not equal that in an interrogator, but this can be dealt with by dividing bias error into two separate source types, one for transponder, one for interrogator. Wherever a common instrument is involved in two range measurements the correlation will be one. Alternately proper fractional correlation coefficients can be assigned between ranges with a common instrument without dividing bias error into two types. The latter is done in the examples which follow, where for illustrative purposes it is assumed that the bias tolerance in both the transponders and the interrogators is the same. In our example, the appropriate correlation coefficient is one half for each of two ranges which have one instrument common to their measurements.

Range scale factor uncertainty is associated with the inability to model index of refraction of the atmosphere precisely. For paths involving a large range of altitude some of the parameters of the atmospheric model are adjusted to match measured meteorological data. The errors in measurement and the failure of the model to match the atmosphere, averaged horizontally at each level, cause the resulting scale factor errors in range measurements to correlate. On the other hand, local variations in the atmospheric index of refraction will not correlate. Some fractional correlation between all $\delta_{A,R}$'s and $\delta_{T,R}$'s is therefore expected. Information is not presently in hand to assert what this correlation should be. Some not unreasonable values have been selected for the illustrative examples as follows:

For two paths to the high flying aircraft, through many level strata of the atmosphere, range scale correlation = 0.4.

For two paths from two ground stations to the low flying test vehicle (nearly horizontal paths over widely different surfaces), range scale correlation = 0.1.

For a nearly horizontal path from one ground station to the test vehicle and an elevated path to the overflying aircraft from a different ground station, range scale factor correlation = 0.1.

For a horizontal path to the test vehicle and an elevated path to the overflying aircraft from the same ground station, range scale factor correlation = 0.2.

When a basis for better correlation coefficients for range scale factor errors is available, then they can be incorporated into the analysis just as these values will be in the examples which follow.

Configuration for the Analysis

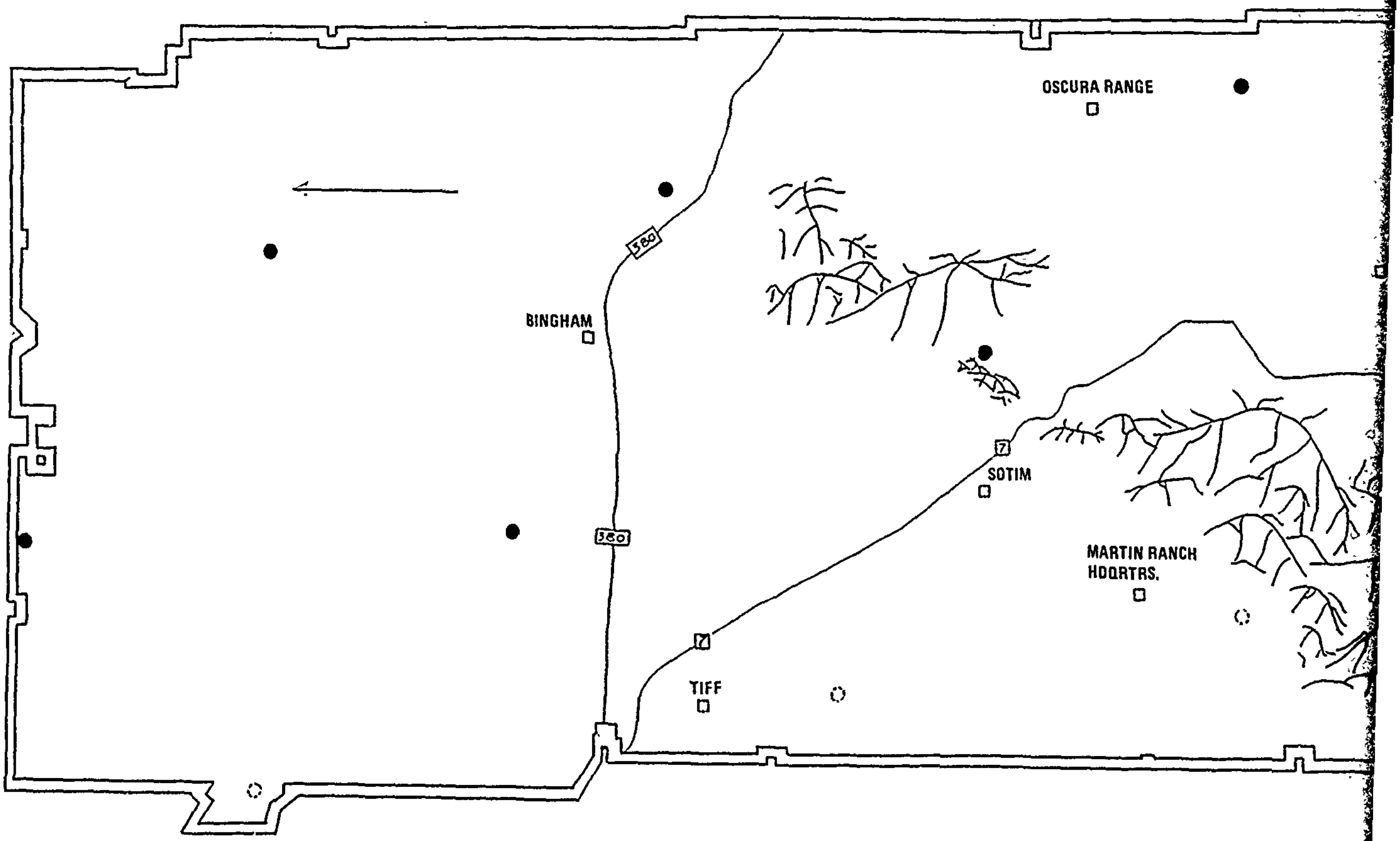
Envisioned is two nominally parallel lines of ground stations, one on each side of a test corridor. To idealize this for analysis the ground stations have all been placed at the same altitude at the apexes of equilateral triangles as in Figure 6. The triangles were chosen to have sides 120,000 feet (20 n.m.) long. An analysis for three ground stations ranging at any one time corresponds to an analysis of the interior of any one of the triangles. Figure 7 is one equilateral triangle in which 5 points, D, E, F, G, H form a suitable collection for evaluating the concept. These 5 points correspond to 4 points every 20 n.m. along a path one third of the way from one row of ground stations to the other row, and likewise along a path halfway between the rows.

The many familar relations associated with these points make the geometrical analysis easy to develop, while adequately illustrating the concepts. Table II lists the ranges to a low-flying target at each point and ranges to an aircraft overflying the point 30,000 feet above. Also, factors that appear in some of the partial derivatives developed above are tabulated.

The Covariance of the Measured Ranges

In order to compute expected uncertainty in the location of aircraft and test vehicle it will be necessary to have expected squares of the uncertainties in the seven measured ranges and the expected products of uncertainty for the 21 pairs of different measured ranges. These covariance elements have been assembled for the specific case of point E, Figure 7, and are laid out in Table III.

Table III consists of 27 rows and 27 columns, associated with the 27 sources of uncertainty. The four (or three, in the case of r_a) sources



OSCURA RANGE
BINGHAM
380
380
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SOTIM
MARTIN RANCH
HDQRTRS.
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TIFF

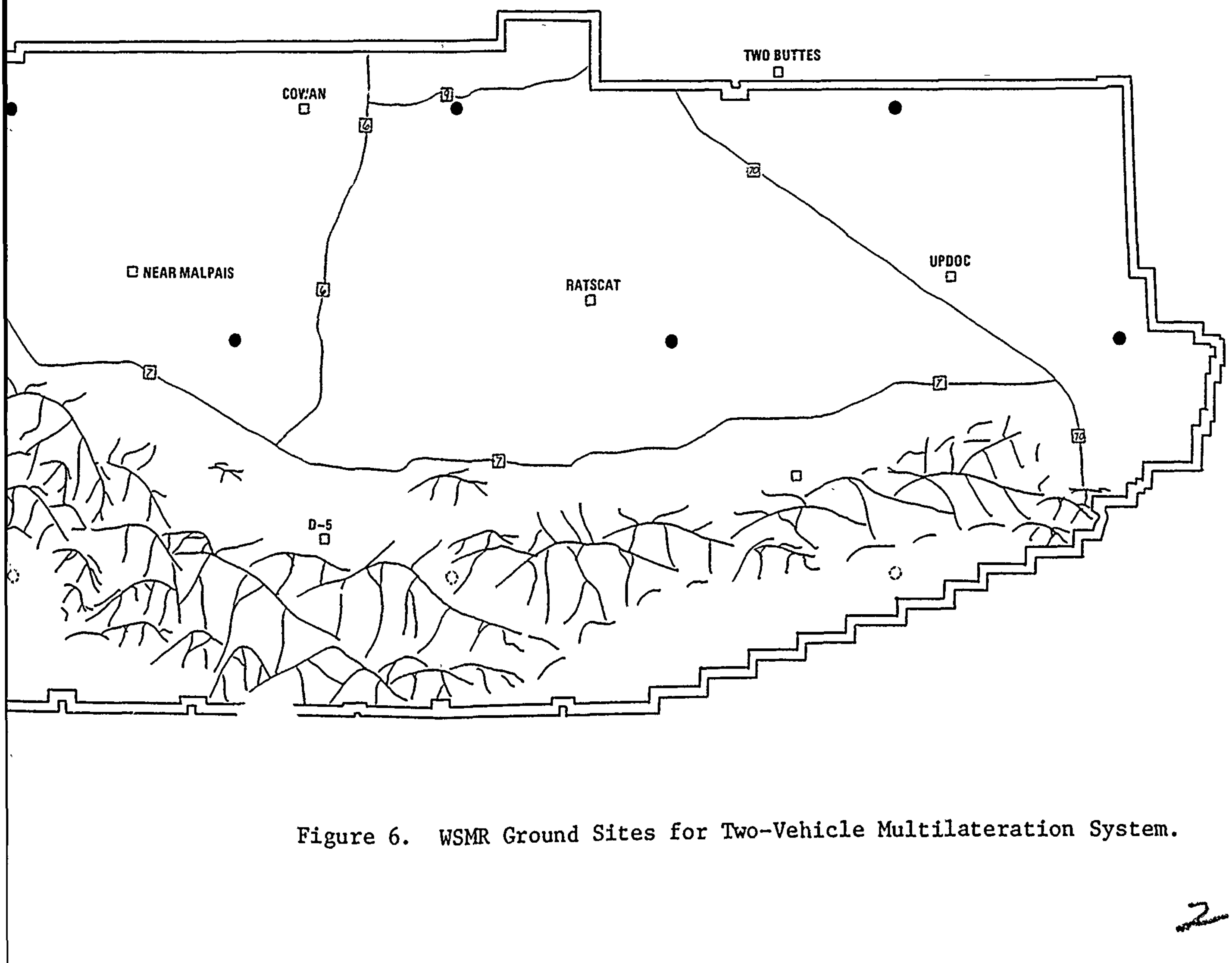

Figure 6. WSMR Ground Sites for Two-Vehicle Multilateration System.

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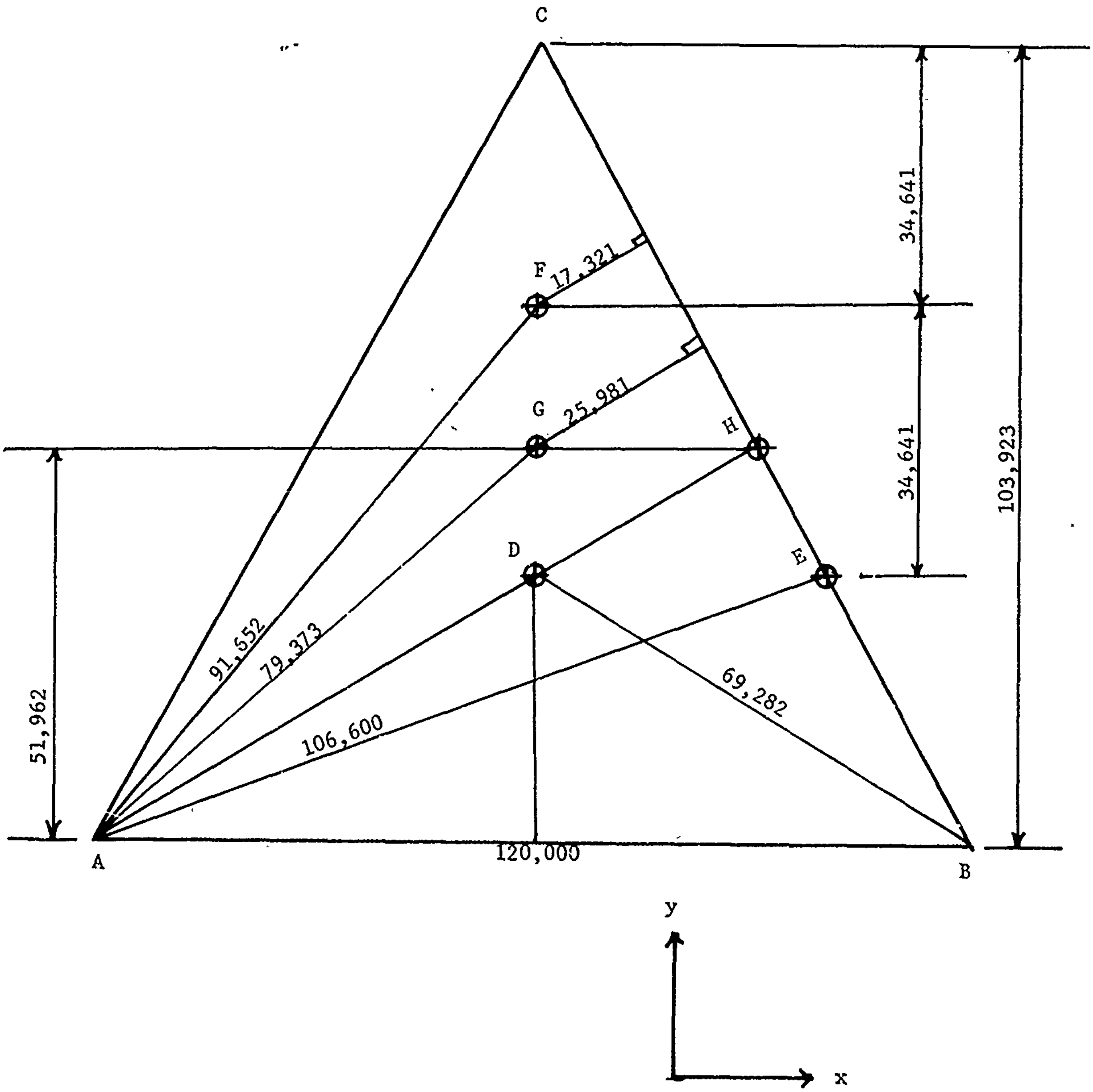

Figure 7. Vehicle Locations Used in Multilateration Error Analyses.

TABLE II

PARAMETERS FOR MULTILATERATION ERROR ANALYSIS

PATH	HORIZONTAL RANGE (ft)	SLANT RANGE (ft)	α (deg)	$\frac{e - X}{e}$	r/e (Target)	r/e (Aircraft)
A-D	69,282	75,498	23.4132	0.3333	0.6667	0.7265
B-D	69,282	75,498	23.4132	0.3333	0.6667	0.7265
C-D	69,282	75,498	23.4132	0.3333	0.6667	0.7265
A-E	105,830	110,000	15.8266	0.0	1.0184	1.0585
B-E	40,000	50,000	36.8699	0.6667	0.3849	0.4811
C-E	80,000	85,440	20.5560	0.3333	0.7698	0.8221
A-F	91,652	96,437	18.1246	0.1667	0.8819	0.9280
B-F	91,652	96,437	18.1246	0.1667	0.8819	0.9280
C-F	34,641	45,826	40.8934	0.6667	0.3333	0.4410
A-G	79,373	84,853	20.7047	0.2500	0.7638	0.8165
B-G	79,373	84,853	20.7047	0.2500	0.7638	0.8165
C-G	51,962	60,000	30.0000	0.5000	0.5000	0.5774
A-H	103,923	108,166	16.1021	0.0	1.0	1.0408
B-H	60,000	67,082	26.5651	0.5000	0.5774	0.6455
C-H	60,000	67,082	26.5651	0.5000	0.5774	0.6455

TABLE

MULTILATERATION RANGE

		r_{A1} N	r_{A1} B	r_{A1} R	r_{A1} S	r_{A2} N	r_{A2} B	r_{A2} R	r_{A2} S	r_{A3} N	r_{A3} B	r_{A3} R	r_{A3} S	N	
r_{A1}	N	9													
	B		9				4.5				4.5				4
	R			1.21				0.22				0.3759			
	S				1.8512										
r_{A2}	N					9									
	B		1/2				9				4.5				
	R			.4				0.25				0.1709			
	S								1.2800						
r_{A3}	N									9					
	B		1/2				1/2				9				
	R			.4				.4				.7300			
	S												1.7534		
r_{T1}	N													20	
	B		1/2												9
	R			.2				.1				.1			
	S				1										
r_{T2}	N														
	B						1/2								1/
	R			.1				.2				.1			
	S								1						
r_{T3}	N														
	B										1/2				1/
	R			.1				.1				.2			
	S												1		
r_a	N														
	B														1/
	R			.4				.4				.4			

Note: Along the diagonal are the variances associated with 27 sources of uncertainty to seven range measurements. The multipath; B bias; R range scale; S survey. A refers to the overflying aircraft, T to the test vehicle and r_a to vehicles. Below the diagonal are the non-zero correlation coefficients between the several sources, and above th values of the uncertainty products.

TABLE III

ATION RANGE ERROR MATRIX, POINT E

	r_{T1}				r_{T2}				r_{T3}				r_a			
S	N	B	R	S	N	B	R	S	N	B	R	S	N	B	R	
																N
		4.5														B
			0.2328				0.0440				0.0880				0.1320	R
				1.9242												S
																N
						4.5										B
			0.0529				0.0400				0.0400				0.0600	R
								1.600								S
																N
										4.5						B
			0.0904				0.0342				0.1367				0.1025	R
7534												1.8727				S
	20															N
		9				4.5				4.5				4.5		B
			1.1200				0.0423				0.0847				0.0317	R
				2												S
					20											N
		1/2				9				4.5				4.5		B
			.1				0.16				0.0320				0.0120	R
								2								S
									20							N
		1/2				1/2				9				4.5		B
			.1				.1				0.64				0.0240	R
												2				S
													9			N
		1/2				1/2				1/2				9		B
			.1				.1				.1				.09	R

easurements. The source labels are N, noise and
vehicle and r_a to the range between these
rces, and above the diagonal are the expected

2

associated with a particular range go with a group of adjacent columns and adjacent rows as the labels indicate.

Along the diagonal of the table are tabulated the squared uncertainties associated with each source. The values are representative of the experience indicated in the many interviews conducted and reports examined during this contract. If specific equipment is contemplated, the uncertainties associated with that equipment should be substituted and the computations carried forward as indicated. If the numerical choices here are deemed appropriate, then the numerical results can be accepted, but a primary purpose of this example is to clearly set down how the calculations should be carried out.

For noise and multipath the squared uncertainty has been chosen to be 9 ft^2 for all ranges at high elevation, i.e., the four ranges involving the overflying aircraft. For the three nearly horizontal ranges, 20 ft^2 has been selected. There is considerable controversy about the magnitude of multipath uncertainty. Clearly, it varies with the terrain, and in any case, the antenna patterns of the several antennas employed are important in limiting its value.

For survey error the figure of 2 feet, vector uncertainty, has been verbally suggested to us for locations on WSMR. If this is taken as 2 feet horizontal, then the component in any one direction will be $\sqrt{2}$ feet. Only one component of the survey error contributes to effective range error. According to (7) this gives 2 $\cos^2\alpha$ ft^2 for the square of uncertainty of ranges to the overflying aircraft and 2 ft^2 for ranges to the test aircraft at nominally zero elevation.

The squared error due to bias has been taken as 9 ft^2 and this includes bias in both the interrogating device and the transponder.

All range scale factor uncertainties are taken as the square of 10^{-5} times the range involved, a figure that seems acceptable to most of the interviewees and report authors.

The range measurements, themselves, are going to be made as unbiased as possible, so the expected value of any range error is zero; but the expected values of the squares of errors (arising out of the 27 sources) are not zero. In an ensemble of range measurements, the individual errors are expected to average zero, but the squares, always positive, will average some positive

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quantity, as is reflected in the diagonal values chosen above. Below the diagonal in Table III have been placed the correlation coefficients discussed above. The blank elements correspond to zero correlation. Above the diagonal are the corresponding expectation value terms, themselves (in ft^2). They are the products of the roots of the corresponding diagonal terms times the associated correlation coefficient. When there is zero correlation the average value of one error term in an ensemble is zero for every specific value of the other, and hence, the average value in the ensemble, that is, the expected value of the product, is zero.

When the correlation is high, as is the case with survey error, every error in the range to one vehicle is proportional to the error in the range to the other vehicle when the ranges are measured from the same ground station. The expected value of the product is the root of the product of corresponding the diagonal terms.

When correlation is partial the correlation coefficient is multiplied by the root of the product of the two diagonal terms. Physically one can envision that the error stems from a sum of "sub-sources," some uncorrelated and some correlated, as was discussed in the case of bias, where part of the error arose in the common instrument, which was fully correlated, while the rest was uncorrelated. Only the correlated part of this sum contributes to the off-diagonal expectation values, the rest averaging zero in an ensemble. In the case of survey error, if the average cosine between the azimuth to the overflying aircraft and the test vehicle is taken as the correlation coefficient, a number close to unity in the configuration being considered, then the component of the survey error of one azimuth, which is parallel to the other azimuth is fully correlated, while the perpendicular component is uncorrelated.

We could work directly with these 27 diagonal members and 36 non-zero off-diagonal members, but it is more convenient to collapse Table III into Table IV. Table IV contains the expectation values of the squares of errors for measurements of each of the 7 ranges along the diagonal and the expected values of the products of the errors above the axis. These turn out to be

TABLE IV

MULTILATERATION RANGE ERROR MATRIX, COLLAPSED, POINT E

r_{A1}	r_{A2}	r_{A3}	r_{T1}	r_{T2}	r_{T3}	r_a	
21.0612	4.7200	4.8759	6.6570	0.0440	0.0880	0.1320	r_{A1}
	19.5300	4.6709	0.0529	6.1400	0.0400	0.0600	r_{A2}
		20.4834	0.0904	0.0342	6.5094	0.1025	r_{A3}
			32.1200	4.5423	4.5847	4.5317	r_{T1}
				31.1600	4.5320	4.5120	r_{T2}
					31.6400	4.5240	r_{T3}
						18.0900	r_a

Note: Along the diagonal are the variances for the seven range measurements. Off diagonal are the expected values of the products of the uncertainties. The matrix is symmetrical, the lower triangle mirroring the upper.

the sums of the terms in the corresponding intermediate sized rectangles in Table III. The reasons for this are as follows:

A differential relation of the form (17) expresses any linear deviation of one of the seven measured ranges. The product of two such relations (including the product of one range differential by itself) expresses how the several error sources combine to form the product. To get the expectation value of an overall product, each product of differentials should be replaced by its expectation value. Thus, in tne product,

$$(\delta r_{A2})^2 = (\delta r_{A2,N}) + 2(\delta r_{A2,N})(\delta r_{A2,B}) + - - - ,$$

only the squared terms on the right have non-zero expectation values according to Table III and the expectation value is,

$$E\left[\delta r_{A2}\right] = \sigma^2_{rA2} = \sigma^2_{rA2,N} + \sigma^2_{rA2,B} + \sigma^2_{rA2,R} + \sigma^2_{rA2,S} \tag{18}$$

the sum of the four diagonal terms associated with r_{A1}.
In the product differential,

$$(\delta r_{A1})(\delta r_{T1}) = (\delta r_{A1,N})(\delta r_{T1,N}) + (\delta r_{A1,N})(\delta r_{T1,B}) + - - -$$

$$+ (\delta r_{A1,R})(\delta r_{T1,R}) + - - - - + (\delta r_{A1,S})(\delta r_{T1,S})$$

only three of the products on the right have non-zero expectation values, and,

$$E\left\langle \delta r_{A1}\ \delta r_{T1}\right\rangle = \sigma_{r_{A1}r_{T1}} = \sigma_{r_{A1,B}\ r_{T1,B}} + \sigma_{r_{A1,R}\ r_{T1,R}} + \sigma_{r_{A1,S}\ r_{T1,S}} \tag{19}$$

Thus, Table IV is composed of the sums of the contents of the numbers inside the intermediate sized rectangles of Table III that are situated above and along the diagonal. This would be the proper rule for collapsing Table III even if there were off diagonal terms within one of these intermediate sized rectangles, provided the off diagonal terms on both sides of the diagonal were included in the sum.

The correlation coefficients associated with Table IV could have been calculated. They are not of great interest since they lack the simple association with the physical system that the coefficients in the lower half of Table III have.

Aircraft Location Uncertainties

Above point E on Figure 7 the differentials of the position of an aircraft are, from (4), (5) and Table II,

$$\delta x_A = \frac{r_{A1}}{e} \cos 30° \ \delta r_{A1} - \frac{r_{A2}}{e} \cos 30° \ \delta r_{A2} \tag{20}$$

$$\delta y_A = \frac{r_{A1}}{e} \sin 30° \ \delta r_{A1} + \frac{r_{A2}}{e} \sin 30° \ \delta r_{A2} - \frac{r_{A3}}{e} \delta r_{A3} \tag{21}$$

$$\delta z_A = \frac{e-X_1}{e \sin \alpha_1} \delta r_{A1} + \frac{e-X_2}{e \sin \alpha_2} \delta r_{A2} + \frac{e-X_3}{e \sin \alpha_3} \delta r_{A3} \tag{22}$$

Thirty degrees is the angle between the x direction and the perpendicular to the lines joining stations B and C and that joining A and C. Signs in (20) and (21) are chosen appropriate to Figure 7. The subscripts on the δr's are 1, 2, 3 standing for the ranges from stations at A, B, C, respectively, in Figure 4.

Numerical values for r/e and (e-X)/e are to be taken from Table II. For all cases in the table, e is the altitude of the equilateral triangle in Figure 4. X is the component of the horizontal range in a direction perpendicular to the line joining the other two ground stations, or is the slant range and α is the elevation angle of the aircraft, which has been assigned the altitude of 30,000 feet.

The "covariance matrix" elements of the aircraft location consist of the expectation values of $(\delta x_A)^2$, $(\delta x_A \delta y_A)$ etc. These are computed by forming appropriate products of (20), (21) and (22) with each other, then replacing products of δr's on the right by their expectation values from Table IV, thus,

$$(\delta x_A)(\delta y_A) = \{(1.0585)(0.8660)\delta r_1 - (0.4811)(0.8660)\delta r_2\}$$

$$\times\{(1.0585)(0.5)\delta r_1 + (0.4811)(0.5)\delta r_2 - (0.8221)\delta r_3\} \tag{23}$$

$$= 0.4851(\delta r_1)^2 + (0.2205-0.2205)(\delta r_1 \delta r_2)$$

$$- (0.7536)(\delta r_1 \delta r_3) - (0.1002)(\delta r_2)^2 + (0.3425)(\delta r_2 \delta r_3)$$

To obtain $\sigma_{x_A y_A}$, the products on the right of (23) are replaced by their expectation values from Table IV. The result is,

$$\sigma_{x_A y_A} = (0.4851)(21.0612) - (0.7536)(4.8759)$$

$$- (0.1002)(19.5300) + (0.3425)(4.6709) \tag{24}$$

$$= 6.1867 \text{ ft}^2$$

The other elements associated with aircraft location at point E are similarly calculated to yield

$$P = \begin{pmatrix} \sigma^2_{x_A} & \sigma_{x_A y_A} & \sigma_{x_A z_A} \\ & \sigma^2_{y_A} & \sigma_{y_A z_A} \\ & & \sigma^2_{z_A} \end{pmatrix} = \begin{pmatrix} 17.4830 = (4.1813)^2 & 6.1867 & -1.8367 \\ & 15.9857 = (3.9982)^2 & -8.7386 \\ & & 52.4231 = (7.2402)^2 \end{pmatrix} \tag{25}$$

The elements of (25) are, indeed, the expectation values of the several products for any one determination of the aircraft's position. The diagonal members are the mean square values of uncertainty in the three coordinate directions. The six elements can be visualized as an ellipsoid inscribed in a rectangular box extending $\pm\sigma_x$, $\pm\sigma_y$, $\pm\sigma_z$ in the three coordinate directions. The orientation of the ellipsoid is given by the three off-diagonal terms.

The equation for the ellipse is

$$\underline{X}^T P^{-1} \underline{X} = 1 \tag{26}$$

where $\underline{X}$ is any radius vector to the surface of the ellipse and P^{-1} is the matrix inverse to P so that $P^{-1}P = I$, the unit matrix. In terms of the six elements of P in (25), the ellipse, (26) is

$$\begin{aligned}(\sigma_y{}^2\sigma_z{}^2 - \sigma_{yz}{}^2)x^2 + (\sigma_x{}^2\sigma_z{}^2 - \sigma_{xz}{}^2)y^2 + (\sigma_x{}^2\sigma_y{}^2 - \sigma_{xy}{}^2)z^2 \\ + 2(\sigma_{xz}\sigma_{yz} - \sigma_{xy}\sigma_z{}^2)xy + 2(\sigma_{xy}\sigma_{yz} - \sigma_{xz}\sigma_y{}^2)xz \\ + 2(\sigma_{xy}\sigma_{xz} - \sigma_{yz}\sigma_x{}^2)\, yz\end{aligned} \tag{27}$$

Its structure is evident if the coordinate axes are rotated to (x',y',z') where P and P^{-1} are diagonal matrices. The new coordinates align with the principal axes of the ellipsoid. In those coordinates, and in terms of the three new non-zero elements of P, (26) and (27) become,

$$\frac{x'^2}{\sigma_{x'}{}^2} + \frac{y'^2}{\sigma_{y'}{}^2} + \frac{z'^2}{\sigma_{z'}{}^2} = 1 \tag{28}$$

The separation of any two parallel planes tangent to the ellipsoid is twice the RMS uncertainty in the direction perpendicular to the planes. The shape of the ellipsoid thereby indicates the relative likelihood of position error in a given direction.

Test Vehicle Location

Our primary interest is in the location of the test vehicle rather than the overflying aircraft. It is located nominally under the aircraft, and for the discussions to follow the elements in Table IV associated with the r_T's and with r_a will be taken to be unchanged for non-zero values of ξ and η, that is, when the test vehicle is located not directly under the aircraft, but displaced horizontally by, say, no more than about 20,000 feet.

Expressions for δx_T and δy_T are exactly like (20) and (21) with T replacing A in the subscripts. The expressions for δz_T is (16) when it is written with δz_A substituted from (22) and $\delta\xi = \delta x_T - \delta x_A$ substituted from (20) and its counterpart for δx_T. Likewise, $\delta\eta$ should be replaced by (21) and its counterpart for δy_T. With these three expressions the six covariance elements of tl target location can be computed analogous to the procedure for the aircraft location. The procedure is straightforward but tedious because of the ralatively large number of terms.

For point E of Figure 7, after entering the appropriate values,

$$\delta x_T = 0.8819\ \delta r_{T1} - 0.333\ \delta r_{T2} \tag{29}$$

$$\delta y_T = 0.5092\ \delta r_{T1} + 0.1925\ \delta r_{T2} - 0.7698\ \delta r_{T3} \tag{30}$$

$$\delta z_T = 1.1111\ \delta r_{A2} + 0.9493\ \delta r_{A3} - \frac{1}{\cos\beta}\ \delta r_a \tag{31}$$

$$+ \frac{\xi}{r_a\cos\beta}\left\{-0.9167\ \delta r_{A1} + 0.4166\ \delta r_{A2} + 0.8819\ \delta r_{T1} - 0.3333\ \delta r_{T2}\right\}$$

$$+ \frac{\eta}{r_a\cos\beta}\left\{-0.5293\ \delta r_{A1} - 0.2406\ \delta r_{A2} + 0.8221\ \delta r_{A3}\right.$$

$$\left. + 0.5092\ \delta r_{T1} + 0.1925\ \delta r_{T2} - 0.7698\ \delta r_{T3}\right\}$$

There is no particular interest in the shape of the error ellipsoid. Its general size is evident from its diagonal terms. As with the aircraft, σ^2_{Tx} is the square of (29) with differential products on the right replaced by expectation values from Table IV. Likewise, σ^2_{Ty} from (30). These lead to

$$\sigma^2_{Tx} = 25.7725\ \text{ft}^2; \qquad \sigma_{Tx} = 5.0767\ \text{ft.}$$

$$\sigma^2_{Ty} = 24.1856\ \text{ft}^2; \qquad \sigma_{Ty} = 4.9179\ \text{ft.}$$

The RMS value of vertical error is much more important to our considerations and σ^2_{Tz} has a form that depends on ξ, η and the value of β that goes with the horizontal displacement between test vehicle and aircraft. β is given by (11) or by,

$$\tan\beta = \frac{\sqrt{\xi^2 + \eta^2}}{\text{altitude of aircraft}} \tag{32}$$

When (31) is squared and the products of δr's on the right replaced by appropriate values from Table IV, then,

$$\begin{aligned}
\sigma^2_{Tz} &= 52.4231 - \frac{0.3279}{\cos\beta} + \frac{18.0900}{\cos^2\beta} \\
&+ \xi' \left(-0.6408 - \frac{4.7933}{\cos\beta}\right) \\
&+ \eta' \left(10.6813 + \frac{0.6130}{\cos\beta}\right) \\
&+ 30.8526\, \xi'^2 \\
&+ 27.9263\, \eta'^2 \\
&+ 23.0751\, \xi' \eta'
\end{aligned} \tag{33}$$

where

$$\xi' = \frac{\xi}{r_a \cos\beta} = \frac{\xi}{\text{Altitude of aircraft}}$$

$$\eta' = \frac{\eta}{r_a \cos\beta} = \frac{\eta}{\text{Altitude of aircraft}}$$

When the test vehicle is directly under the aircraft, $\xi = \eta = 0$, and $\beta = 0$, and

$$(\sigma^2_{Tz})_{\xi=\eta=0} = 70.1852\ \text{ft}^2; \quad (\sigma_{Tz})_{\xi=\eta=0} = 8.3777\ \text{ft}$$

When the test vehicle is not directly under the aircraft, σ_{Tz} increases. It is desirable to know how far from the overhead position the aircraft can be without having σ_{Tz} exceed some specified value. One can insert such a value into (33) and compute ξ' as a function of η' (or vice-versa) to find the horizontal extent of the region where the vertical measurement will be in some sense satisfactory. This will reveal how closely the aircraft must locate over the test vehicle.

To the extent that β is constant in (33) with (σ_{Tz}^2) fixed, the solution represents an ellipse in the (ξ',η') plane. The term $18.0900/\cos\beta$ varies rapidly enough with ξ' and η' that it is not profitable to discuss this ellipse. Rather, (32) and (33) can be solved directly for a series of values of one variable. Figure 8 consists of plots for $\sigma_{Tz} = 10$ ft. Inside the contours, σ_{Tz} is less than 10. The points on the contour correspond to the dimensionless value of ξ' and η' multiplied by the altitude of the aircraft, 30,000 feet in this example. The closest approach of the contour of σ_{Tz} to the origin is about 0.65, which amounts to 19,500 feet for the aircraft at 30,000 feet, Point E.

Point D, Figure 7, has its symmetry reflected in the resulting covariance elements. All off diagonal terms are zero, and,

$$\sigma_{Ax} = 3.5062 \text{ ft.}$$

$$\sigma_{Ay} = 3.5062 \text{ ft.}$$

$$\sigma_{Az} = 7.9200 \text{ ft.}$$

$$\sigma_{Tx} = 4.2375 \text{ ft.}$$

$$\sigma_{Ty} = 4.2375 \text{ ft.}$$

Right under the aircraft,

$$(\sigma_{Tz})_{\xi=\eta=0} = 8.9644 \text{ ft.}$$

The contour of horizontal positions for which σ_{zT} = 10 feet is a circle of radius 0.7110, in the (ξ',η') plane, which amounts to 21,330 feet or 3.56 nm. Data for the other three points of Figure 7 are contained in Table V and Figure 8.

Discussion of Lateration Results

Correlations. One striking item that came out in the foregoing analysis is the very strong correlations between some of the error sources. The use of an instrument common to more than one range measurement is the source of correlation between certain bias terms and likewise, between station survey errors. The magnitude of these two sources of correlation is clear; the choices of correlation coefficients in the examples are appropriate, and the adjustments to match a particular system in which bias in the transponders may have a different expected value from those in the querying devices are straightforward.

Correlation associated with range scale uncertainty is less clear. The choices in the examples are not unreasonable ones, but no experience is in hand to guide the choice of the correlation coefficients. Certainly the correlation exists. There is a temptation to take comfort in the expected correlations of range scale, through a layered, partially known atmosphere, up to an overflying aircraft and downward to a test vehicle. Indeed, this correlation tends to cancel the uncertainty in the altitude of the test vehicle, but examinations of Table III will reveal that range scale error is one of the smaller sources of uncertainty. The bias correlation between the three ranges up to the overflying aircraft is also strong, and pertains to a source of greater uncertainty. This correlation accentuates the uncertainty in the altitude of the overflying aircraft, and through this, the altitude of the test vehicle. On balance the correlations result in greater uncertainty in the altitude of the test vehicle.

The correlations result in smaller variances in horizontal directions. The correlated lengthening (or shortening) of the three ranges to either vehicle due to bias in the transponder carried by the vehicle and the correlation of scale uncertainty is responsible for this.

TABLE V

COVARIANCE MATRICES OF AIRCRAFT LOCATION AND TARGET LOCATION AT POINTS E THROUGH H IN FIGURE 7

(Target Directly Under Aircraft)

Point F

$$\underline{\underline{A}} = \begin{pmatrix} 20.4948 & 0 & 0 \\ & 10.9626 & 6.2512 \\ & & 44.9828 \end{pmatrix}$$

$$\underline{\underline{T}} = \begin{pmatrix} 31.7935 & 0 & 0 \\ & 14.9603 & -1.3653 \\ & & 62.7128 \end{pmatrix}$$

Point G

$$\underline{\underline{A}} = \begin{pmatrix} 15.6817 & 0 & 0 \\ & 10.6071 & 3.1174 \\ & & 58.4212 \end{pmatrix}$$

$$\underline{\underline{T}} = \begin{pmatrix} 23.6881 & 0 & 0 \\ & 21.8434 & 1.3788 \\ & & 76.0793 \end{pmatrix}$$

Point H

$$\underline{\underline{A}} = \begin{pmatrix} 18.5140 & 5.1424 & -5.8005 \\ & 12.5758 & -3.3481 \\ & & 61.8212 \end{pmatrix}$$

$$\underline{\underline{T}} = \begin{pmatrix} 27.9476 & 8.3921 & -5.1130 \\ & 18.2595 & -2.9522 \\ & & 79.5512 \end{pmatrix}$$

Point E (Lasers)

$$\underline{\underline{A}} = \begin{pmatrix} 10.6116 & 4.1229 & -3.9975 \\ & 10.3005 & -5.3753 \\ & & 21.3209 \end{pmatrix}$$

$$\underline{\underline{T}} = \begin{pmatrix} 9.5882 & 4.2082 & 0.0632 \\ & 9.3818 & -0.0370 \\ & & 29.1282 \end{pmatrix}$$

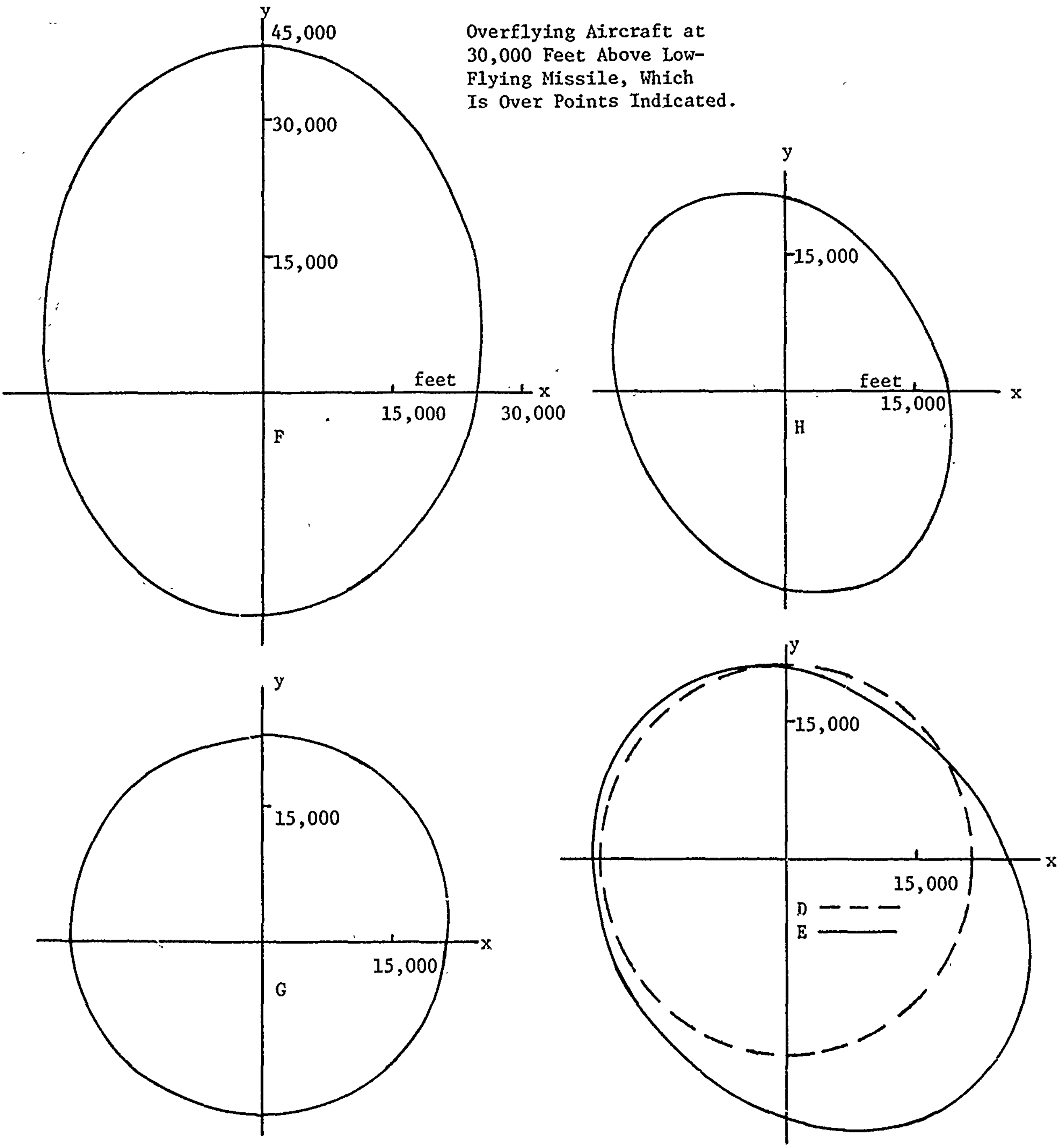

Figure 8. Bounds on Overflying Aircraft Position for 10-foot RMS Vertical Error--Radio Ranging.

The variances of the three aircraft location coordinates about point E are repeated in Table VI along with the values that result if all correlations are ignored. Likewise, similar variances are given for the test vehicle directly under the aircraft.

Comparison of the numbers in Table VI suggests that better knowledge about the range scale correlations is not going to dramatically change the numerical results of these examples.

TABLE VI

MULTILATERATION MATRIX DIAGONAL (POSITION) ELEMENTS, WITH AND WITHOUT CORRELATIONS

Coordinate	Variances, σ^2, and Their Roots, σ			
	Correlations Included		Correlations Ignored	
X_A	17.4830	4.1813	21.0881	4.5922
Y_A	15.9857	3.9982	20.8747	4.5689
Z_A	52.4231	7.2404	42.5697	6.5245
X_T	25.7725	5.0767	28.4435	5.3332
Y_T	24.1856	4.9179	28.2325	5.3134
Z_T	70.1852	8.3777	60.6597	7.7884

Magnitudes of Sources of Range Uncertainty. In the examples the values chosen for range uncertainties are representative of state-of-the-art, or at least what the state-of-the-art is thought to be by the persons interviewed and the authors whose papers were read for this study. The firmness of the several values differs, however, and deserves some discussion. The detailed methods of computation here set down can, of course, be applied to any revisions of the values of range error sources.

There is extensive experience with the effects of circuit noise and the stability of trigger circuits. The effects of these on the behavior of a timing instrument can be and have been measured under controlled conditions, so the effects of noise and instrument bias are probably accurate.

The range scale error would be three parts in 10^4 if no allowance were made for the presence of the atmosphere at sea level. A reasonably good model of the atmosphere that does not change with time should reduce the uncertainty to three in 10^5; and if meteorological data is skillfully applied, one part in 10^5 is to be expected. Certainly the range scale error need not be as large as three in 10^5, and it is unlikely that it can be kept to three in 10^6.

Site survey error is not usually as small as two feet over a test range extending 50 or 60 miles each way from its middle. WSMR is very special in this regard, and the choice may well be realistic.

The examples in this analysis dealt only with horizontal survey errors. Vertical errors can be simply included and how to do so was laid out. In any event, it is the expected component of survey error parallel to the range vector that will enter the computations.

The most controversial of the sources is multipath, particularly at lower elevation angles, as envisioned for determining the horizontal position of the test vehicle. The consequences of the low angle multipath variance (20 ft^2 including instrument noise, but not instrument bias) being too small should be understood. Certainly it will increase the horizontal uncertainty of the test vehicle, but this system of location was analyzed with the notion that the horizontal location specifications for the test vehicle could often be relaxed if only its altitude uncertainty could be maintained.

If the overflying aircraft is directly over the test vehicle, then the horizontal uncertainty of the test vehicle does not enter into the evaluation of its altitude uncertainty. This can be seen from Equation (16). There the last two terms are,

$$\frac{\xi\delta\xi}{r_a\cos\beta} + \frac{\eta\delta\eta}{r_a\cos\beta} = \frac{\delta(\xi^2+\eta^2)}{2\text{ (Aircraft Altitude)}} = \frac{\delta h^2}{2\text{ (Aircraft Altitude)}} \tag{34}$$

$$= \frac{h\ \delta h}{\text{Aircraft Altitude}} = \tan\beta\ \delta h$$

When β is small, the effect of uncertainty in the horizontal position, h, of the test vehicle relative to the aircraft does not affect the altitude determination. From Table V the altitude variance z_T at Point E for this condition is 70 ft^2, while the horizontal variance of h is of the order of

$$\sigma_{Ax}^2 + \sigma_{Ay}^2 + \sigma_{Tx}^2 + \sigma_{Ty}^2 \sim 67 \text{ ft}^2$$

Now to keep σ_{Tz} within 10 feet, the contribution of the term in (34) must not exceed 30 ft^2. That is,

$$\tan^2\beta \; (67 \text{ ft}^2) < 30 \text{ ft}^2$$

$$\beta < 33.8 \text{ degrees.}$$

This is inexact, neglecting some correlations, but it gives a fair estimate of the size of the contour in Figure 8. The closest approach of the contour to the origin corresponds to β = 34.2 degrees, the furthest to 48.2 degrees.

The consequence of less favorable multipath conditions on the ranges to the low-flying vehicle is that the overflying aircraft must maintain its position over the test vehicle more accurately to keep its altitude determination within limits. This follows even if the increased horizontal position uncertainty can be tolerated.

Signal coding in the ranging equipment can possibly reduce the effect of multipath on range measurements. The interaction is complex, and not widely understood. With any equipment proper attention to the antenna will reduce the non-direct energy that is the multipath reflection. The Cubic [10] antenna has been configured to concentrate the gain above the horizon for the ground stations. The siting of all antennas should be given adequate consideration; and at some sites the antennas may have to be very carefully designed to minimize multipath errors.

Likewise, the siting of ranging antennas on the vehicle is important. On the test vehicles, particularly, it may be impractical to measure radiation patterns and do all that is literally possible, so the best skill and intuition should be employed.

Usefulness of the System. Examination of the computed data at the several points examined indicate that the uncertainty in locating the test vehicle does not vary radically from point to point inside the equilateral triangle. The precision with which an overflying aircraft has to stay with the test vehicle also is reasonably constant. On-line estimation of the test vehicle's position relative to the aircraft will usually have to be used to maintain station, either through presentation to the pilot, or to an autopilot.

The requirement that the aircraft stay over the test vehicle restricts the precise measurements to a small portion of WSMR for any one test flight, or to test vehicles that the aircraft can follow, which may mean restriction to subsonic test flights.

The original goal of 10 feet RMS uncertainty in each axis can be met by this system even when the aircraft is not directly over the test vehicle. This analysis was carried forward in the belief that the altitude of the test vehicle might often be the really critical measurement. It has been shown that multilateration would permit altitude measurements with 10 foot accuracy, with the overfly conditions in Figure 8.

There are two refinements that could improve the performance of the system. At most positions over WSMR more than three ground stations can range on the test crafts. Redundant data, if properly employed, will improve the results; but this would be a minor improvement, especially on the altitude of the test vehicle. When the additional data is most likely to be available is near the cross-over between one triangle and the next, that is, near points E or H of Figure 7. At E or H the computed altitude of the overflying aircraft depends entirely on ranges from stations B and C. Range from A, or from the fourth station, the third corner of the other triangle containing B and C, can add little to the precision of the altitude of the aircraft.

Improvement in the two horizontal estimates will enhance the test vehicle altitude estimate when it is not directly under the aircraft. However, at points interior to the triangle, like D, G, or F. Figure 7, range to a fourth station is greater than 20 n.m.; and the signal-to-noise ratio will at such a range (depending on designs of the ranging devices) begin to degrade the information. Redundancy would probably be of little value.

More important than redundant range data is smoothing of the computed trajectory. The foregoing analysis addresses only a single position determination by seven range measurements. In practice, the seven measurements will not be simultaneous, and computation of the position at any one time must take this into account. If a high data rate can be maintained, then it will be practical to smooth the trajectory and still keep the details of the actual motion of the vehicle. Smoothing will average down the effects of noise, multipath, and small scale fluctuations of the atmospheric refractivity. It cannot change effects of bias, station survey, or large scale, slowly changing uncertainties in the atmospheric refractivity. This averaging can be significant since noise and multipath constitute a large fraction of the range measurement uncertainty.

Reduction of multipath error would enhance the system accuracy. If the ranging system is a pulse leading edge system, with pulses on the order of 10 nanoseconds, and with peak power increased to maintain the energy in each pulse high, the ranging error due to signal reflections could be reduced.

Multilateration with Laser Ranging Devices

There is presently a proliferation of laser ranging devices being developed for military and non-military needs.

A review of the errors in the above analysis of a radio ranging, multilateration system indicates several ways that a laser based multilateration system could improve accuracy in determining the position of a low-flying test vehicle.

Multipath error could be greatly reduced by virtue of the enhanced ratio of desired (retroreflected) to undesired (stray reflected and scattered) signal. Furthermore, the laser pulse length can easily be held to less than 10 nanoseconds.

Equipment delay uncertainties would be reduced because the "transponders" would be retroreflectors.

The scale factor error would be reduced at least by 10 percent, and a two-wavelength method of measuring atmospheric refraction effects might permit further reduction [42].

The errors of a laser-based system can be estimated by assuming the following error source effects:

Noise and multipath: 4 ft^2, all ranges.
Bias: 4 ft^2.
Range scale factor: $0.81 \times 10^{-10} r^2 ft^2$.
Survey error: $2 \cos^2 \alpha \, ft^2$ to aircraft.
$2 ft^2$ to target.

The correlation values may be assumed to be the same as in the radio ranging analysis, except for the correlation of bias uncertainty. Four separate lasers and range measuring devices could be mounted in the overflying aircraft, and one at each ground site; or alternatively and more practically, two lasers and range measuring devices could be located at each ground site, and one in the aircraft. The absence of a common bias delay in different range measurements reduces the bias correlation to practically zero.

Using the above values, with the target and aircraft both located directly over Point E the position errors are:

$\sigma_{Ax} = 3.2575$ ft. $\sigma_{Tx} = 3.0965$ ft.
$\sigma_{Ay} = 3.2094$ ft. $\sigma_{Ty} = 3.0630$ ft.
$\sigma_{Az} = 4.6175$ ft. $\sigma_{Tz} = 5.3971$ ft.

Figure 9 is a plot of the "overfly" contour -- the ground projection of the limit within which the aircraft must hold its overflying pattern to keep the target altitude error within 10 feet RMS. The closest point of the contour to Point E corresponds to a value of the look-down angle (measured from the vertical downward direction) of 50.4 degrees, which means that the overflying aircraft can stray at least as far as 36,300 feet from directly over the low-flying vehicle, without causing the error in estimation of the altitude of the low-flying vehicle to exceed 10 feet RMS.

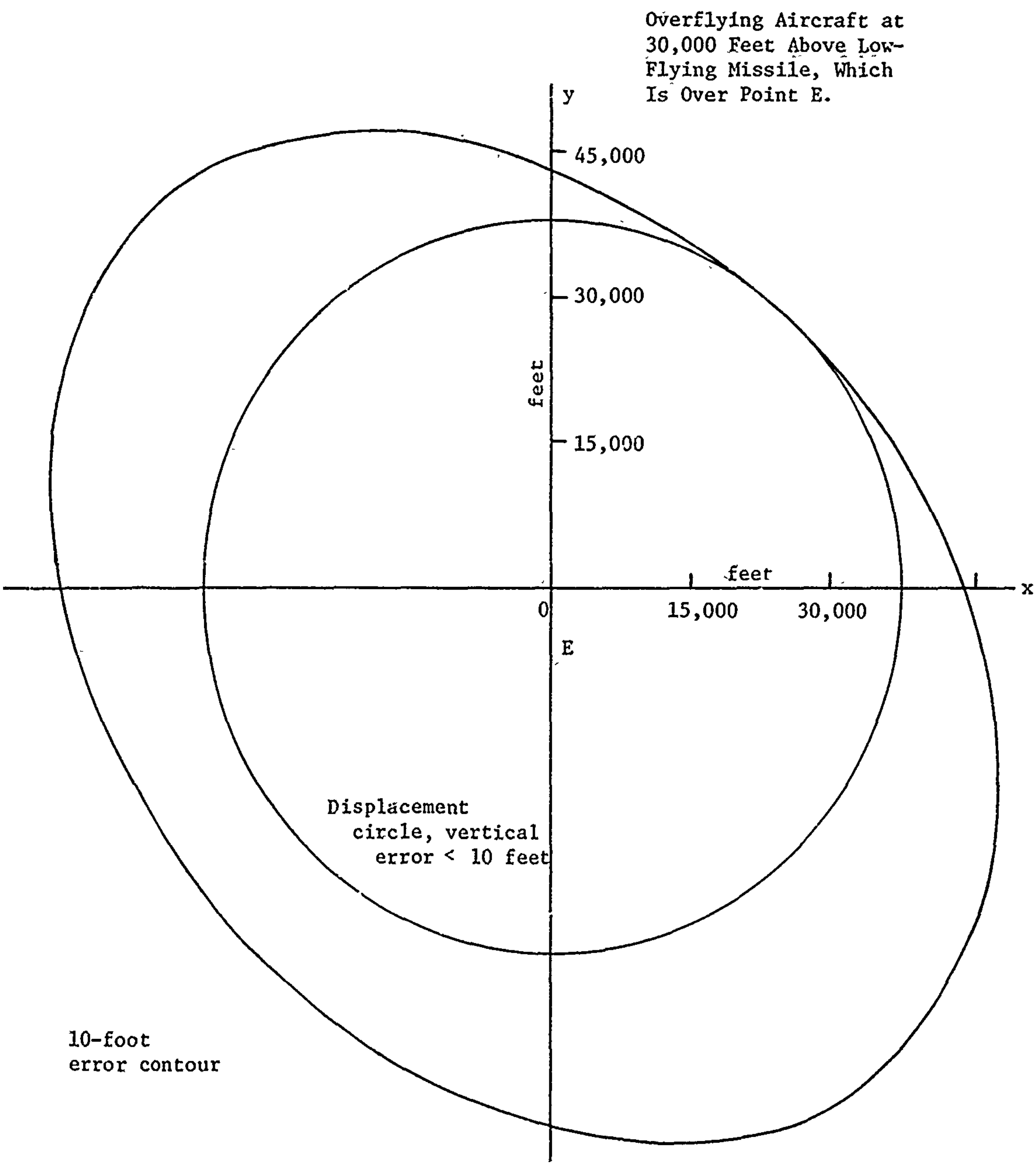

Figure 9. Bounds on Overflying Aircraft Position for 10-foot RMS Vertical Error--Laser Ranging.

5. CONCLUSIONS

The primary conclusion that has been reached in this research is that the system shown in conceptualization in Figure 1 is feasible for precision measurements of position, velocity, and acceleration of low-flying missiles and aircraft at White Sands Missile Range. The goal of the contract, however, was to find available equipment for a system like that in Figure 1 which would enable WSMR to make precision measurements. Part of this system is available; part would have to be developed. That part which is available has been termed in this report a CIRIS-type system. A CIRIS-type system includes ground-based reference transponders which are positioned with very high accuracy through ground survey (accurate to two parts per million). On board the high altitude aircraft is a radio range measurement set, a high-accuracy inertial measurement unit, a barometric altimeter, and a computer. The radio reference system makes measurements of range and range rate from the ground based transponders. The inertial measurement unit estimates position and attitude of the airborne platform from its gyroscopes and accelerometers. An estimate of altitude of the airborne platform is derived from a barometric altimeter. These estimates of position, velocity, and acceleration are combined or computed by an "optimum" algorithm in the digital computer.

A CIRIS-type system exists at Holloman Air Force Base, and is reported to have met its original specifications [36]. The conclusion reached in this report is that this system, with a sufficient number of ground based transponders spaced in square grids 80,000 feet on a side, would be capable of determining the position of the airborne reference platform within about 5.8 feet RMS, any axis. The attitude of the reference platform can be determined within about 22 arc seconds.

No airborne radar or other system was found, however, which would permit range and pointing angle measurements from the airborne reference platform to the low-flying missile with sufficient accuracy to meet the overall specifications of 10 feet RMS position, any axis, for the low-flying missile. The conclusion is that such a system is feasible; however, it would have to be developed. The R&D program necessary to develop the airborne radar (which would have to operate at 70 GHz or 95 GHz) would be a major undertaking. The radar transmitter, the antenna, the radome, and the pointing mechanism and circuitry, as well as the transponder which would be mounted in the target missile, would have to be developed, and prototypes would have to be constructed. Georgia Tech has had experience in the development of both 70 and 95 GHz radar systems, but these were not airborne systems. They were designed for ground vehicles such as armored personnel carriers and surface effect vehicles.

One alternative to the use of conventional radar in the link between airborne platform and target, as shown in Figure 1, would be a laser radar. This alternative has been analyzed in this report. The laser could be mounted in boresight with a 17 GHz radar. The latter would serve to acquire the target, and the laser radar component would lock on and track a retroreflector mounted on the low-flying missile.

Another alternative concept would be to track the low-flying target entirely from ground based positions using an available ground based laser radar system, PATS, which is manufactured by Sylvania. PATS was observed in action at Yuma Proving Grounds. In the test at Yuma, a helicopter was tracked at a distance of about 36,000 feet with an estimated error of about 5 feet RMS. PATS consists of a YAG laser which is boresighted with a telescopic video camera. The laser and camera tube are mounted elevation over azimuth. The video display on a closed circuit television screen is used to acquire the target, through joystick control of the laser radar mount. When the target is approximately centered on the video display screen, the operator transfers control to the automatic tracking mode of the laser radar. Accuracy specifications for PATS is 0.1 milliradians in each of the two axes, elevation and azimuth, and two feet in range up to 65,000 feet. Maximum range for this system is said to be 100,000 feet. Some nine or ten PATS laser radars could track low-flying targets over most of White Sands Missile Range. The elevation angle of a laser radar can be depressed below horizontal because the reflectivity of the earth is much, much less than the reflectivity of the retroreflector mounted on the target. Multipath is thus no problem with laser radar, except perhaps over water. The effect of atmospheric index of refraction on angle measurement error has not been defined, however.

The laser radars could also be considered as replacements for the WSMR theodolite cameras, which are presently the basic instrumentation for the range. The laser radar units are capable of making measurements at low altitudes whereas the theodolites are not. The laser radars also permit real time or almost real time data turn-around, whereas the theodolites require days or even weeks for data turn-around.

A third alternative system, which would employ existing Ku-band radar, has also been described in this report. The airborne reference platform would have to maintain station within 8 degrees of vertical over the low-flying target. The down-looking radar would serve to measure range to the target. So long as the angle of the line-of-sight from reference platform to target is within 8 degrees of vertical, the overall error in <u>vertical</u> position of the target would be within 10 feet RMS. The reference platform would be positioned by a CIRIS-type system described in Chapter 2. The horizontal error would be on the order of 40 feet, because of the low frequency of the Ku-band radar. The usefulness of this concept is that existing equipment, with some modifications, could be used. The assumption is that the most important position coordinate of the target is altitude.

The fourth concept which has been examined is a total range measurement, or multilateration concept. In this scheme range (and range rate) would be measured from ground based sites to the target, as well as to the airborne reference platform. In the measurements from ground sites, only the information concerning the horizontal position coordinates of the target would be retained; the altitude data would be discarded because it would be known to be inaccurate. The range and range rate measurements from the airborne platform (40,000 feet AGL) to the target would be the source of information for the estimate of target altitude.

The analysis of the multilateration approach indicated that the low-flying target position could be determined with an error less than 10 feet RMS, any axis, using radio ranging measurements. Extension of the analysis to laser ranging devices indicated that errors of 5 feet RMS, any axis, may be feasible.

6. RECOMMENDATIONS

The need for adequate instrumentation to measure the performance of low-flying missiles and aircraft is unquestionable. It was found that the Figure 1 system cannot be fully implemented with available equipment; the airborne radar conceptualized in Figure 1 would have to be a 70 GHz or a 95 GHz radar or a laser radar. Analyses were made of the system requirements of airborne millimeter and laser radars, which would have to be developed.

In addition to three airborne radar, four other systems were analyzed. All seven potential systems are listed in the decision matrix, Table VII. Of the seven, only PATS is immediately available, and its cost is high. Furthermore, the pointing angle error for PATS low-angle tracking has not been established. However, a PATS system could replace ciné theodolites, giving WSMR immediate data turnaround. PATS would probably extend WSMR measurement capability to altitudes lower than the theodolites can handle.

The development cost of the multilateration system which has been analyzed in this report would be relatively small. Indeed, it is believed that a number of moderate improvements could be made in the RMS/micro B equipment that would reduce errors to levels that would permit better than 10 foot RMS position accuracy in a multilateration system. A multilateration system using laser ranging might permit position determination within 5 feet.

A complete system design study of a multilateration system is recommended. Both radio and laser ranging would be examined. Various interrogator/transponder configurations, and the consequent telemetry and data reduction needs would be evaluated.

If there are positive results from the multilateration study, prototype ranging equipment would be developed, and an experimental, abbreviated, multilateration system would be implemented and tested.

In the event that the system design study of the multilateration system indicates the approach is not feasible, it would be recommended that attention be shifted to 95 GHz and laser radars to be used in conjunction with an airborne platform and ground-based position determining system. First a review would

TABLE VII

DECISION TABLE FOR RECOMMENDATIONS

DECISION FACTORS	ALTERNATIVE PROGRAMS: MULTILATERATION						
	Radio Ranging	Laser Ranging	CIRIS/TRACKER: Laser	95 GHz	70 GHz	Ku Band	PATS Laser Radar
Order of Recommendation	1	2	3	4	5	7	6
Reasons for Rankings	Lowest R&D and system cost Variation of index of refraction along LOS causes more position uncertainty through errors in angle measurement than through errors in range measurement.	Hightest potential accuracy	Increasing beamwidth error → No water vapor refractive effect			Limited to altitude measurements.	Highest system cost. Pointing angle error has not been determined.
Counteracting Factors	Terrain sensitive: requires LOS from 3 ground sites to test vehicle (and to overflying aircraft) over most of the flight.					Lowest R&D cost of airborne trackers	Could replace theodolites. Fast data turnaround.

be made of airborne laser trackers, to determine the R and D gap between existing equipment capabilities and the WSMR requirements. An assessment would then be made of relative cost-benefit of a prototype laser, versus a 95 GHz, radar.

A prototype airborne tracking radar would then be developed, and incorporated in a purchased CIRIS-type system (ARIS, for example). An abbreviated system of ground sites would be implemented, and the system would be tested.

Regardless of which system reaches prototype realization--a multilateration or a CIRIS/airborne tracker system--the next stage of the recommended program would be a range-wide system design. It would include system control, telemetry network, data processing, and other system features. The resulting design would be a basis for requests for bids from manufacturers for a complete system which would enable WSMR to measure performance of low-flying missiles and aircraft.

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APPENDICES

APPENDIX A

LOW ALTITUDE TRACKING PROBLEM DEFINITION
(WSMR Internal Memorandum)

Background

US Army White Sands Missile Range has a long standing need to provide trajectory measurements on targets that sustain flight at low altitude. The requirements were summarized and documented in 1967, (reference 1). Low level intrusion is a very attractive offensive and counter offensive tactic. The Department of Defense has invested much time and money in recent years to develop guidance and control technology for low level intrusion weapon delivery systems. These developments have included: high quality inertial guidance systems, terrain matching systems, and terrain avoidance control systems. The success of these developments has intensified the need to develop instrumentation that can be used to evaluate weapon systems employing the new technology. Experience with complex weapon systems in the Vietnam conflict has shown that testing in a more realistic environment is required to assure operational effectiveness. Weapons that worked well under benign test conditions failed completely in combat. This indicates that low level intrusion weapons need to be tested in an environment approximating that which would be encountered in actual deployment. The development of such a capability at USAWSMR would not duplicate a function provided by any other DOD test facility. The development of this unique capability for USAWSMR should enhance position of the Range by providing a new capability that is needed by all services and not available at alternate locations.

Measurement Environment

In considering the measurement environment it is assumed that USAWSMR will cooperate to the fullest extent possible with providing test data under realistic conditions consistent with safety requirements. This implies that the measurement environment should duplicate the distances and types of terrain that might be encountered in actual combat situations. The actual combat environment can be visualized by considering targets which US Forces might be required to engage using low level intrusion techniques. In the present world situation, desert terrain, mountainous terrain, wooded hills, and jungle areas are all likely areas where weapons of this type might be deployed. Some objectives might require flights over hundreds of miles across varied terrain. The response of a weapon delivery system to such an environment cannot be adequately tested by short flights over level terrain. Hence the required measurement environment for low altitude tracking is a large area with varied terrain. Vehicles that sustain flight at low altitude must be relative large in order to carry sufficient fuel to complete the flight. This means that the vehicle is large enough to carry a transponder.

Measurement Accuracy

A survey of the projects currently assigned to USAWSMR, that require low altitude tracking, was conducted to determine the measurement accuracy need for present weapons system technology. It is recognized that the measurement accuracy requirements stated in the UDS are not always a completely accurate statement of needs. It does represent the only official record of what is needed and is the basis for committment of USAWSMR resources for testing. The survey indicated that 29 test programs (see Table A-1) currently being conducted at USAWSMR require support for sustained flights below 10,000 feet. The following summarizes the existing requirements for low altitude flight measurements.

UDS Low Altitude Requirements Summary

Minimum Altitude Flown	0-200 ft	200-500 ft	500-2000ft	2000-10,000 ft
No. of Projects	8	10	2	7

Maximum Range Flown	10-20 mi	20-50 mi	50-100 mi	>100 mi
No. of Projects	6	3	5	9

Position Measurement Accuracy Required	1-5 ft	5-10 ft	10-50 ft	>50 ft
No. of Projects	11	6	8	2

Velocity Measurement Accuracy Required

	.05-1 ft/sec	1-5 ft/sec	5-10 ft/sec	>10 ft/sec
No. of Projects	6	12	2	7

Acceleration Measurement Accuracy Required

	.1-1 ft/sec^2	1-10 ft/sec^2	10-50 ft/sec^2	>50 ft/sec^2
No. of Projects	6	6	3	1

Table A-1

Current Projects Having Low
Altitude Flight Measurement Requirements

UDS#	NAME	UDS#	NAME
75	MQM-61	486	ASM TEST
89	SAM-D	520	PAVE DEUCE
105	FAAR	521	AQM-34U TERCOM DEMO
124	HAWK	379	A/C IN NAV SYS
147	NV 123	420	LORAINS
151	MQM 34D	449	AF-EMP
152	TALOS LAST	492	621B FIELD TESTS
157	HITVAL	495	DEFENSE SUPPRESSION
158	MODEL 1089	518	INHI FLT TEST
160	YAQM-37A	522	BI NAV TEST
301	HAWK/HIP	713	CEFIRM LEADER
364	BQM-34A	808	NAV AIR WPNS TEST
374	HOUND DOG	833	F 14 FLT TEST
452	MAVERICK	953	EMP SIM
464	SRAM		

The projects used in compiling this summary are identified in Table A-1. Comparison of the above summary with similar summaries compiled in 1967 (reference 1) and 1970 (reference 2) indicate that these requirements have remained at a consistent level for the past six years. If USAWSMR is to respond to the need for more realistic testing, it is imperative that a capability for meeting these requirements be provided. It is suggested that the following be established as design goals for a trajectory measurement system to meet this need:

Coverage: Provide data on targets flying 200 feet AGL over the USAWSMR area with future expansion capability to include the USAWSMR-Green River corridor.

Position Measurement Accuracy: 10 feet

Velocity Measurement Accuracy: 5 feet/second

Acceleration Measurement Accuracy: 5 feet/second^2

Data Output: Digital data available for use in flight control and flight safety applications.

Operational Environment

The system purchased to meet the need for low altitude tracking should provide for simple reliable operation at reasonable cost. It should be assembled from proven component subsystems not requiring research or further development apart from system integration. Ease of maintenance and calibration are important attributes for the system and should be prominent factors in system design. The personnel requirements for system operation should be minimized consistent with maintaining reasonable system cost.

Summary

This report has provided a discussion of the background and present requirements for an instrumentation system that furnished low altitude flight measurements. The information included indicates that the need for such a system has existed for at least the past six years. The problem is expected to continue for the foreseeable future. The report also shows that a low altitutde tracking capability would provide USAWSMR with a unique testing asset.

(Signed)

ROBERT E. GREEN
Mathematician

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(Appendix A)

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APPENDIX B

AIRBORNE TRACKING SYSTEM
FOR
LOW ALTITUDE TRACKING
(WSMR Internal Memorandum)

I. Introduction

USAWSMR has an established need for a measurement system to provide trajectory data on vehicles that sustain flight at low altitude. The requirement is for a system that can provide data on vehicles flying 200 feet AGL anywhere on the Range. The required trajectory data accuracies are:

Position: 10 feet

Velocity: 5 feet/second

Acceleration: 5 feet/second^2

The vehicles tested under these conditions will carry beacons or transponders as tracking aids. The purpose of this report is to describe a tracking system that could be procured to meet these requirements. The proposed system will be described along with its operation, expected accuracy, and estimated cost. Possible alternate uses for the system or its components will also be discussed.

II. Airborne Tracking System Description

The proposed configuration for an airborne tracking system consists of three major components:

An airborne platform location system.

A small phased array radar.

A test vehicle transponder.

The first two items are to be mounted in a pod that can be attached to an aircraft using standard Air Force pod hangers if possible. The test vehicle transponder will be mounted in the object being tested while flying at low altitude. The airborne platform location system will be used to locate the position of the instrumentation pod being carried on a high flying aircraft while the phased array radar will track the transponder on the test object from the pod. The suggested equipment configuration consists of the following.

A. Airborne Platform Location System

The airborne platform location system proposed configuration includes a range and range rate measuring system and a high quality inertial navigation system. This configuration provides two independent estimates of aircraft position with different types of error statistics. This should provide better information than can be achieved by combining two systems with similar error statistics. The range and range rate system will be required to produce accurate measurements that can be processed for real time display. The required performance is two feet accuracy in range and .1 foot/second in range rate sampled five times per second. The recommended configuration for the range and range rate measurement system is an airborne interrogator that provides simultaneous measurements to at least four ground transponders. This type of system is recommended for the following reasons:

1. Studies performed by the Air Force indicate that system accuracy cannot be met unless simultaneous measurements are performed (1).
2. Airborne tracking systems of this type have been fabricated for similar applications (2).
3. This configuration allows the measurement of Doppler velocity without the need to transmit a reference frequency between ground stations. This eliminates a major source of error and expense for Doppler measurement systems.
4. Present technology allows such a system to be packaged for airborne application.
5. Measurements to four ground stations provide some redundancy for greater reliability and error estimation.

6. The range rate measurements can be used to significantly reduce the noise content of the range data thereby enhancing the accuracy of platform location. The velocity information is required to provide update information for the inertial navigation system. This results in much improved accuracy from an inertial navigation system(1).

The inertial navigation system should be one that is presently in the operational inventory for DOD aircraft. This obviates the need for development in an area where the Range has very limited expertise. The inertial navigation system will play a dual role in the airborne tracking system. The data from it will be used to estimate the position of the airborne platform and the orientation of the phased array radar. The inertial navigation system will also be interfaced with the phased array radar for an altimeter input.

B. A Small Phased Array Radar

The small phased array radar will be used to look down from the airborne platform to track a transponder equipped target flying near the ground. The phased array radar approach is chosen since its electronic agility eliminates the need for three-axis stabilization required for a mechanical tracking device. The multiple target tracking capability of the phased array eliminates the need for a separate altimeter in the system and also makes in-flight calibration of the radar system practical. The radar can be used as an altimeter by directing a beam down from the airborne platform. The inertial navigation system can be used to determine the downward direction. In-flight calibration of the radar can be accomplished by locating additional radar transponders at the location of the range and range rate measurements system transponders. Interrogation of these transponders should provide accurate inflight calibration of the phased array radar system. It is recommended that the radar operate at <u>K-band</u> using an array with an aperture of <u>approximately 36 inches</u> in diameter. It is suggested that the radar be equipped with a four target capability for the required operational flexibility.

Each track channel should provide for either beacon or skin tracking. The present Air Force inventory of airborne phased array radars should be investigated to determine if available equipment could be adapted to meet this need.

C. A Test Vehicle Transponder

The test vehicle transponder is used to separate the tracked target from the radar echo returned from the ground and to increase the tracking range capability of the system. A conventional K-band radar beacon should provide the desired target enhancement for tracking in the low altitude environment. For this application, the antenna should be mounted on top of the test vehicle to provide the required coverage and limit the amount of power illuminating the ground. Transponders for this application should be readily available from industry.

III. System Operational Concept

A test conducted using an airborne tracking system will require an aircraft to carry the instrumentation pod and a test vehicle transponder mounted in the test object. The flight of the instrumentation aircraft and the test object must be coordinated so that the separation between the two does not exceed the tracking range of the phased array radar. The ground transponders are placed along the flight path of the instrumentation aircraft in a pattern that minimizes errors due to system geometry. It is suggested that the instrumentation aircraft be operated at an altitude of approximately 40,000 feet. The Air Force maintains a fairly large inventory of aircraft that can be operated at this altitude. The suggestion for mounting the system in a pod that can be carried by whatever aircraft is available is an attempt to avoid being restricted to a single plane that may not be available when needed. In an actual test operation the instrumentation aircraft will be flown over a prescribed course to coincide with the launching of the test object. The platform location system will be used to determine the position of the aircraft. The phased array radar will then acquire and track the test vehicle. The relative location of the two aircraft at the beginning of the test will be a function of the relative speeds that are flown. If the instrumentation aircraft can fly at approximately the same speed as the test vehicle, then the test might begin with the test vehicle slightly ahead of the instrumentation aircraft. If the test vehicle flies much faster than the instrumentation aircraft then the instrumentation aircraft would be positioned ahead of the test vehicle in order to maximize the amount of time that the test vehicle will remain within range of the radar. Initial acquisition techniques will require further investigation. Possible alternatives include the use of the aircraft bomb sight by the pilot, calibrated to direct a search by the phased array radar. Initial acquisition might also be provided from information generated by ground based instrumentation. It is suggested that the data processing performed in the airborne tracking system be limited to that required for effective operation and control. The remainder of the data processing can be performed by the USAWSMR UNIVAC 1108 computing system. The requirement to mount the equipment in a pod necessitates minimizing size and weight of the airborne equipment. It is suggested that the data generated by the airborne tracking system be transmitted to the ground for processing using standard telemetry equipment. The telemetry system is designed for reception of such data and provides for direct entry into existing computing facilities. The use of the radar as an altimeter requires that the altitude of the point measured to on the ground must be known. The presently available maps of the USAWSMR area should provide sufficient accuracy for this purpose. It is suggested that a task be initiated to develop a method of reducing the map information to digital data for use with an airborne tracking system.

IV. Range and Range Rate Transponder Deployment

The optimum elevation angle for a range and range rate tracking system is approximately 35 degress (3). Using this criterion, twelve transponders deployed on the Range would provide very good geometry over the entire area for an aircraft flying at 40,000 feet altitude. The system can be used with the transponders spaced further apart resulting in somewhat lower accuracy. The inertial navigation system can be used to provide data when the system is in unfavorable geometric locations. It is suggested that the initial system procurement acquire four transponders with the remaining eight being acquired after the system has been acceptance tested.

The following suggested transponder locations have been chosen from maps of the USAWSMR area and represent a typical deployment. Inspection of these sites may indicate that some are not operationally suitable. The proposed site locations are:

A. Four station optimal geometry for checkout.
   1. UPDOC
   2. TWO BUTTES
   3. CHUCK
   4. 3.5 miles west of SW 30

B. Four station Range wide coverage.
   1. UPDOC
   2. COWAN
   3. D-5
   4. SOTIM 3

C. Twelve station range wide coverage.
   1. UPDOC
   2. TWO BUTTES
   3. CHUCK
   4. 3.5 miles west of SW 30
   5. COWAN
   6. D-5 (on Gunsight Peak or Salinas Peak)
   7. Along RR9 17 miles east of SALINAS
   8. SOTIM 3
   9. MARTIN Ranch

10. OSCURA RC

11. TIFF

12. Intersection RR9 and Highway 380

It is expected that a narrow corridor from USAWSMR to Green River can be instrumented with these twelve transponders. Further analysis is required to determine the best transponder deployment for this application.

V. System Error Budget

The following system error budget is estimated based on available information. It should be recognized that this information is preliminary and will be refined as the system is more completely defined.

A. Range and Range Rate System

1.	Range measurement accuracy	2 feet
2.	Range rate measurement accuracy	1 ft/sec
3.	System position measurement accuracy	1.5 feet

B. Inertial Navigation System

1.	Position measurement accuracy	.5 foot
2.	Attitude measurement accuracy	10 $\widehat{\text{sec}}$

C. Altimeter System

1.	Radar altimeter measurement accuracy	5 feet
2.	Map location and height accuracy	3 feet

D. Airborne Platform Location System

Estimated accuracy of the position and attitude obtained by combining A, B, and

Position accuracy	2 feet
Attitude accuracy	10 $\widehat{\text{sec}}$

E. Phased Array Radar System

1.	Range measurement accuracy	5 feet

2. Angle measurement accuracy 20 sec

3. Position measurement accuracy relative to the airborne platform for slant range to a target of 100,000 feet. 10 feet

F. Geodetic Measurement Systems

Position measurement accuracy 2 PPM

G. Airborne Tracking System

Position measurement accuracy for slant range to target of 100,000 feet. 10 feet

It is expected that position data of the quality indicated can be differentiated to provide velocity and acceleration data of the quality specified for the system.

VI. System Cost Estimate

The cost estimate furnished here is preliminary and should be replaced by a more careful and detailed engineering cost estimate.

A.	Range and Range Rate Tracking System One Airborne interrogator and four transponders	$1,000,000.00
B.	Inertial Navigation System	200,000.00
C.	Phased Array Radar	1,500,000.00
D.	Systems Integration and Packaging	500,000.00
	TOTAL	$3,200,000.00

VII. Alternate Uses

The acquisition of an airborne tracking system would enhance the USAWSMR capability to support testing in other areas besides low altitude tracking. The ability to use the system for other applications increases the system utilization and permits the amortization of system cost over a greater percentage of the testing workload. The Airborne Tracking System or components thereof could be applied to the following USAWSMR testing problems:

Aircraft flight testing

Range instrumentation calibration

Air to air missile testing

Low altitude drone control

Near launch missile tracking

The first three items of this list are applications for using the system as configured for low altitude tracking. For aircraft flight testing, the system could be attached to the vehicle being tested. The normal operation of the airborne platform location system would perform the required function. As indicated earlier, the transponders could be redeployed to cover greater distances. The lower accuracy achieved would be sufficient to meet many user requirements. The Airborne Tracking System could function as a standard for Range instrumentation calibration. The accuracy specified for the airborne platform location system is sufficient to identify bias errors in present Range instrumentation. The use of such a system for calibration should improve the performance of present instrumentation by providing a tool that can be used to reduce bias errors significantly. An airborne tracking system would provide a cost effective method of calibration since some calibrations could be performed when the system was being used as test instrumentation. It could be used for calibration of most types of USAWSMR instrumentation and the cost of special calibration flights would not be excessive. The Airborne Tracking System should provide a significant improvement in Range capability to support air-to-air missile launches and intercepts. This is particularly true for high altitude tests of these small sized missiles. If the system can be successfully pod mounted, it might be possible to carry this instrumentation on the missile launching aircraft. Operation of the phased array radar at the short ranges involved should provide good quality data for scoring air-to-air intercepts. An airborne tracking system is also potentially useful as a device for controlling drones flying at low altitude. The system could be interfaced with the Vega drone control system to provide the required control functions. This application requires that the original system be modified by adding the Vega control system. The last suggested alternate use of the proposed system is for near launch missile tracking. This application would require only the phased array radar portion of the system. The small size and electronic agility of the radar make it possible to locate the equipment near a missile launcher. The radar could provide data very early in a missile flight. The data provided could be used for flight safety monitoring, direction of other instruments, and metric measurement data. The application of an Airborne Tracking System for these uses in addition to low altitude tracking indicate that it is a cost effective solution to the problem.

VIII. Requirements Summary for Alternate Uses

A brief summary of requirements for the four alternate uses identified is included to show that the Airborne Tracking System can be useful in meeting these needs.

A. Aircraft Flight Testing Measurement Accuracy Requirements:

Function	Accuracy Required		
	Highest	Lowest	Median
Position	2.0 ft	50.0 ft	37.5 ft
Velocity	0.10 ft/sec	5.0 ft/sec	1.1 ft/sec
Acceleration	10.0 ft/sec^2	15 ft/sec^2	12.5 ft/sec^2
Attitude	0.05 deg	1.0 deg	0.53 deg

Aircraft flight testing represents a significant portion of the USAWSMR workload. It is estimated that 20 per cent of the workload is aircraft flight testing.

B. Range Instrumentation Calibration

This function deals not with amount of workload but with quality of results furnished to Range customers. The required calibration accuracy levels for each major measurement instrumentation system is included in this table.

Function		Accuracy Required		
Instrument Type	Contraves	Askania	DOVAP	Radar
Position	1.0 ft	1.0 ft	2.1 ft	9 ft
Velocity	1.2 ft/sec	1.0 ft/sec	0.10 ft/sec	1.0 ft/sec
Acceleration	1.6 ft/sec^2	1.2 ft/sec^2	0.20 ft/sec^2	5 ft/sec^2
Attitude	1.0 deg	1.0 deg		

C. Air-to-Air Missile Testing Measurement Accuracy Requirements:

Function	Accuracy Required: Highest	Lowest	Median
Position	1.0 ft	10.0 ft	5.0 ft
Velocity	0.20 ft/sec	5.0 ft/sec	1.0 ft/sec
Acceleration	1.0 ft/sec^2	16.0 ft/sec^2	3.20 ft/sec^2

This category represents a fairly small number of Range customers, but is usually afforded a high priority due to its importance to the defense effort.

D. Low Altitude Drone Control Requirements

The Range has not yet been requested to provide data for the low altitude drone control function. It is estimated that to control a drone flying 200 feet AGL, that data accurate to 50 feet would be required.

E. Near Launch Missile Tracking Measurement Accuracy Requirements:

Function	Accuracy Required: Highest	Lowest	Median
Position	0.15 ft	100 ft	5.0 ft
Velocity	0.10 ft/sec	100 ft/sec	5.0 ft/sec
Acceleration	0.01 ft/sec^2	32.0 ft/sec^2	7.5 ft/sec^2

This category also represents a significant portion of the USAWSMR workload. It is estimated that 30 per cent of the Range customers require near launch tracking data. Data is presently being provided using Fixed Cameras. This method of data collection is slow and expensive. If the phased array radar could be used for half of these projects, the saving would be significant in both time and money.

IX. Summary

This paper has presented a description of an Airborne Tracking System as a proposed solution to USAWSMR low altitude tracking problem. The system proposed will meet the requirements stated in the problem statement. The Airborne Tracking System can be used to instrument flights over hilly and mountainous terrain as well as over flat terrain. It appears that the system can be expected to provide the required measurement accuracy. Besides providing the required capability for low altitude tracking, the Airborne Tracking System equipment could be applied to other Range measurement problems. Hence an Airborne Tracking System would provide a workable cost effective method of meeting the USAWSMR low altitude tracking requirements.

APPENDIX C

REQUIREMENTS FOR A SYSTEM TO MEASURE PERFORMANCE OF LOW-FLYING MISSILES AND AIRCRAFT

The following outline specifies the WSMR requirements for a system to measure low-flying test vehicles:

I. General Requirement:

An instrumentation tracking system is required to provide accurate trajectory data on test vehicles flying at low-altitudes anywhere on White Sands Missile Range.

II. Target: (Test Vehicle)

A. Type: Missiles, RPV's, A/C, etc., but probably typified by SRAM (cruise missile).

B. Number: Single target.

C. Velocity: Both subsonic and supersonic targets must be considered.

D. Altitude: 200 feet to 1000 feet AGL typical.

E. Expected RCS: 5 - 15 dBsm typical.

F. Target Instrumentation: Radar transponder.

III. Coverage/Operational Scenario:

A. Area: Test vehicle located anywhere on (over) WSRM; future expansion to include Green River Corridor.

B. Terrain: Desert & mountainous.

C. Weather: Clear with low humidity; little or no rain.

IV. System Error Requirements:

A. Position Measurement Accuracy: 10 feet, any axis.

B. Velocity Measurement Accuracy: 5 feet/second.

C. Acceleration Measurement Accuracy: 5 feet/second^2.

V. Data Format and Processing:

A. Format: Standard telemetry equipment compatible.

B. Computation Equipment: IBM 360/65 (on-line with telemetry system) and UNIVAC 1108.

C. Philosophy: Utilize ground-based processors to maximum extent possible.

VI. Airborne Instrumentation:

A. Test Vehicle Target: Limited to transponder (relatively small package).

B. Other A/B Instrumentation: Packaged in a standard bomb-rack pod, weighing no more than approximately 1000 pounds, and having dimensions of 15 feet long by 3-foot diameter cylinder. Pod should be completely interchangeable between aircraft.

C. Availability: Instrumentation currently within the military/commercial inventory should be used to the maximum extent possible.

D. Maintenance and Calibration: Prime considerations.

VII. Ground-Based Instrumentation:

A. Mobility/Transportability: Equipment should be as small and transportable as possible consistent with other system constraints.

B. Unattended Operation: Ground-based instrumentation may be required to operate at remote locations and unattended.

C. Survey Error: On the WSMR, ground-based instrumentation can be located to an accuracy of 2 parts per million.

D. Availability: Instrumentation currently within the military/commercial inventory should be used to the maximum extent possible.

E. Maintenance and Calibration: Prime considerations.

VIII. Proposed System Configuration:

A. Primary Components:

1. Position location system consisting of at least 4 ground-based transponders, an airborne interrogator, data processor, and an inertial navigation system.

2. Airborne instrumentation platform (probably an aircraft).

3. Airborne radar capable of acquiring and tracking the test vehicle target.

4. Radar transponder on-board the test vehicle.

B. Operation: By providing measurement of range and range rate (nominally) between the A/B interrogator and 4 ground-based transponders, the position measurement system establishes an estimate of the position of the airborne platform. A second, independent estimate of position is obtained from the inertial navigation unit carried on-board the airborne platform. Combining the two independent position estimates improves the overall position estimate. Position data on the test vehicle relative to the airborne platform is obtained from the A/B radar measurements of range and angle. Radar attitude stabilization is obtained from the inertial navigation unit also. The transponder on board the test vehicle target improves the received S/N and allows the target return to be separated from the ground return.

C. Specific System Parameters:

1. A/B platform altitude is approximately 40,000 feet AGL.
2. A/B platform velocity is approximately the same as test vehicle.
3. Slant range from A/B platform to target more than 40K feet but less than 100K feet.
4. Expect A/B radar to operate above X-band.
5. Elevation angle for position location system should be optimized to approximately 35°.

APPENDIX D

POSITION LOCATING SYSTEMS

The following outline is a condensation of the characteristics of a number of position locating systems. It includes systems with ranging only, as well as systems combining range and inertial measurement. One system is a laser radar which measures azimuth, elevation, and range to the target from a ground based site.

I. AROD (RRS)

A. Data Source/References (all by Motorola):

1. AROD Test and Feasibility Demonstration Program Definition.
2. AROD Vehicle Tracking Receiver Design.
3. AROD System Concept.
4. AROD System Test Model.
5. AROD Test Model Hardware.
6. AROD Flight Demonstration Proposal.
7. AROD Flight Demonstration Test Report [12].

B. Operational Description:

1. Three or more ground-based, completely automatic transponders.
2. Space vehicle based interrogator.
3. Space vehicle based computer.
4. Range modulation: $\pm$ 90° phase shift, PN code.
5. Readout: 4/sec.
6. Acquisition: 2 sec.
7. PN code: Low clock rate for acquisition; high clock rate for tracking.

   Length: 6.084×10^6 count equivalent.

   Down link: 2.214 GHz.

   Up link: 1.800 GHz.

   Command: 137.5 MHz.

   Transponders: 60.

   S-Band: 20W.

VHF: 6W.

Threshold: -126 dBm.

Dynamic range: 27 dB.

N. F.: 8.3 dB

Power Required:

Interr: 143W.

Trans.: 220W.

Tracking BW: Range, 4-5 Hz; carrier, 200 Hz.

Signal: As strong as -70 dBm degrades the performance.

C. Employment (Scenario):

Range and range-rate measurements from space vehicle to ground are transmitted on a turn-around S-band link. Range is determined from two-way time delay; range rate, from Doppler shift of S-band carrier frequency. PN code length assures no ambiguity within 3.042×10^6 m. Transponders are phase locked loop tracking type--not easily adapted to multiple interrogators.

Interrogation of three transponders is simultaneous, while fourth is being acquired. Pick-up and drop are automatic, controlled by range.

D. Principal Sources and Magnitudes of Error:

1. Range to position geometrical blow-up error (GDOP) 10 times the range error.
2. Survey error 1×10^{-5}.
3. Altitude measurement error (negligible if calibrated during line crossing).
4. Equipment error 0.7 feet RMS.
5. Atmospheric propagation velocity error, 6×10^{-6}.

E. Accuracy Specifications (bench test):

Range

Resolution 0.25m
Accuracy $\pm$ 0.5m (0.75m, with 26 dB co-channel interference)
R max, 2×10^6m (unambiguous range)

Range Rate

Resolution 0.02 m/s
Accuracy $\pm$ 0.015 m/s
$\dot{R}$ max $\pm$ 1.2×10^4 m/s
$\ddot{R}$ max 450 m/s^2 (for 20 sec)

F. Cost Estimate:

10's of thousands of dollars for transponders (1967).

G. Availability: No working system exists.

II. CIRIS, Litton/Cubic CR-100 (RRS, IMU, Kalman)

A. Data Source/References:

1. CIRIS Design Evaluation Report [1].
2. Precision Ranging System, CR-100 brochure.
3. Study of Instrumentation Methods for Precision Determination of Aircraft Position, Velocity and Attitude [2].
4. Telephone conversations with:
   (a) Richard Pearson, Holloman AFB.
   (b) Bard Crawford, TASC.
   (c) Visit to Litton and Cubic.
5. Post-Flight Filtering and Smoothing of CIRIS Inertial and Precision Ranging Data [4].

B. Operational Description:

1. Radio Reference System: Ground based transponder (R and $\dot{R}$), Cubic CR-100.
2. Airborne interrogator.
3. IMU: Litton AN/ASN-86, with Navigation Computer Unit, which uses barometric altimeter input to vertical channel.

C. Employment:

Sequential interrogation from airborne reference platform of the ground site transponders, at rate of 5 sec per transponder, 15 sec for three units. Range and range rate are obtained at same time in each interrogation. Dropout of one transponder and pick-up of another, to get optimum location accuracy, is possible. The

RRS may be viewed as reinitializing the IMU, or the IMU can be viewed as a smoothing filter to give continuity of data between RRS interrogations. A 10-state Kalman filter permits the hybrid system to be more accurate than either component (IMU or RRS) alone, provided the filter is properly designed. This implies good prior knowledge of the characteristics of the sources of error.

D. Principal Sources of Error:

1. IMU sensors (gyros and accelerometers).
2. Attitude readout.
3. Range measurement (scale factor and atmospheric disturbances).
4. Range rate measurement.
5. Barometric measurement.
6. Survey.
7. Computer mechanizations.

E. Accuracy Specifications:

Position: 12.5 feet RMS (150 mile maximum spacing between transponders)

Velocity: 0.05 ft/sec RMS

Attitude: 15 $\widehat{\text{sec}}$/axis RMS

F. Cost Estimate:

$100,000 for each transponder (space shuttle version)

$1,500,000 for airborne unit

G. Generic system is operational at Holloman AFB.

III. AC Carousel/Cubic CR-100 (RRS, IMU, Kalman)

A. Data Sources/References: Same as II.

B. Operational Description:

1. RRS: Cubic CR-100, range only, ground based.
2. Airborne interrogator.
3. IMU: AC Carousel IV, with 32-speed resolver for azimuth readout.
4. Northrup NDC-1051A computer.

C. Employment:

Same as II, except that Kalman filter has 22 states (including 3 for survey errors when in "survey mode"). Every fifth measurement is the output of an altimeter, and four transponders are interrogated cyclically rather than three. Every 10 seconds the system is updated by a single scalar measurement, so 50 seconds is the period of an interrogation cycle.

D. Principal Sources of Error:

Same as II, but with bias tip rate in place of azimuth gyro scale factor error and certain other sensor errors. Absence of range rate information as an independent measurement affects error distributions and magnitudes.

E. Accuracy Specification: Same as II.

F. Cost Estimate: Same as CIRIS/Litton/Cubic.

G. Generic system is operational at Holloman AFB.

IV. SHIRAN (RRS)

A. Data Source:

1. Motorola report [2].
2. SHIRAN Geodetic Survey System, Electronic [13].

B. Operational Description:

1. 3 GHz.
2. 4 of 6 transponders at a time.
3. 500 miles capability.
4. Preflight calibration (pole beacon).
5. Interrogation: 12 millisec, each station, every 0.1 sec.
6. 4 sinewave frequencies, lowest gives least significant digit = 500 miles.
7. ϕ modulation $\pm$ 12 rad.
8. RF BW = 35 MHz.
9. Continuous range tracker, 10 samples/sec input, 22 bits @ 5/sec (110 bits/sec) output.

10. Receiver: -107 dBm sensitivity.
11. Transmitter power: 20W, airborne and ground units.
12. Antenna gain: A/C, 8 dB; ground, 18 dB.
13. Transponder: 250 lb. 50 foot pole mounted, must be monitored.
14. Dynamic range: not designed for short transmission paths.

C. Employment Scenario:

Aerial surveying. Calibration by pole beacon; line crossings for range and altitude calibration during flight. Adaptable to multiple users and to slaving.

D. Principal Sources of Error:

1. Atmospheric effect of index of refraction along propagation path.
2. Accuracy of survey of benchmarks used for reference.
3. Calibration errors.

E. Accuracy (measured):

1. Position resolution: 9 inches.
2. Position accuracy: 3 m (includes propagation and survey error).

F. Cost Estimate:

$25,000 for each of six transponders.

$200,000 for airborne interrogator.

G. Availability: Exists, has military designation, AN/ASQ-32.

V. PLRS Hughes/Gen. Dynamics, (RRS)

A. Data Source/References: Notes [11].

B. Operational Description:

1. One master unit, one sub-master unit.
2. Many man-packed, surface vehicular, and airborne units.
3. Range measurements.
4. Trilateration computes three dimensional position.
5. Unit display of position, navigation, related information.

6. Time slot reporting.
7. 100 message types.
8. 1.875 second reporting cycle (frame).
9. 9 millisecond range/message time slot, 900 per frame.
10. Aircraft reporting cycle: 2 seconds at 15 per second maximum rate for a mix of users.

C. Employment:

Tactical data support system for command and control of deployed amphibious assault forces. Capacity: 370 users.

D. Principal Error Sources:

Probably equipment, since accuracy specifications are poor.

E. Accuracy Specifications:

Zone A	Az	El
slow, fixed wing a/c	50 m	50 m
high speed	200 m	200 m
Zone B		
slow	200 m	200 m
high speed	400 m	400 m

F. Cost Estimate: Several million dollars for a full system.

G. Availability: Operating system at Navelex, Fleet Marines.

VI. RMS-2/DCS (Range Measuring System/Data Collection System), General Dynamics

A. Data Source/References: Notes, brochure [7].

B. Operational Description:

1. Fixed and mobile interrogation (A units).
2. Relay (D units).
3. One centralized, computer interfaced (C unit).
4. Range [C/A or D:9km; A/B:64km(LOS)]. By command from C unit, A unit interrogates B unit by sending a ranging pulse, measuring time to response, sending 15-bit number to C unit.

5. Time Slot: 0.744 percent duty cycle/B unit.
6. WWV synchronization.

C. Employment:

Cylinder 20 miles diameter, 20,000 feet altitude. Men, vehicles, aircraft. Position and communication. C unit uses semi-trailer (10 tons) scaffold tower, parabola and omni antennas. Full computer/terminal equipment. Power required: 18 kW. A-station has erected tower and unmanned electronics.

D. Principal Sources of Error:

1. Survey errors for C and A units.
2. Propagation errors.
3. Equipment errors, A/B units, including A unit clocks.

E. Accuracy Specifications:

1. Position, ± 3 meters, with respect to known reference, in x, y, z coordinates.
2. Precision of ranging: ± 2 meters.
3. Clock must thus have pulse jitter less than ± 7 nanoseconds.
4. ± 20 meters reported as experienced Yuma Proving Ground.

F. Cost Estimate:

C unit: $350,000
A unit: $ 50,000
Micro B unit: $35,000 each

G. Availability: Operational at Yuma Proving Ground.

VII. RMS/SCORE, General Dynamics (RRS, IMU, Kalman)

A. Data Source/Reference:

1. Trip reports.
2. Brochures [8].

B. Operational Description:

Same as item VI, RMS-2/DCS, with additions of SCORE (Simulated Combat Operations Range Equipment), large scale computer capability and large screen 3-D real time display. SCORE has an aircraft sub-system which includes:

IMU (strapdown)
Signal conditioner
Micro B transducer
Antenna and radome
Air data unit

C. Employment:

Extends RMS-2/DCS from primarily locating ground based equipment and low-flying support aircraft to include high-flying aircraft.

D. Principal Sources of Error: IMU, and same as in item VI.

E. Accuracy Specifications:

Position: 25 feet any axis

Velocity: 15 feet/sec.

IMU:

Accelerometer bias (3σ): 2×10^{-3}g.
Accelerometer misalignment (3σ): 205 $\widehat{\text{sec}}$.

Flight test errors (in good, transitional, and bad geometry regions):

X and Y: $\pm$ 4 meters
Z: $\pm$ 6, 8, 10 meters
Roll and pitch: $\pm$ 1 degree
Yaw: 1.5, 2.0, 2.5 degrees

F. Cost Estimates:

SCORE pod: $100,000

Micro B unit: $35,000 each
C unit: 350,000 each
A unit: 50,000 each

G. Availability: Can be ordered.

VIII. ARIS (Airborne Range Instrumentation System) Litton (RRS, IMU, computer)

A. Data Source/Reference:

1. Trip report.
2. Brochure [5].

B. Operational Description:

SUU - 16, gun type pod, 22 inches diameter, 15 feet long, 800 pounds.

IMU: AN-92 INU.

Computer: ASN-92 ANCU

Pitot tube probe.

Air pressure transducer.

Interrogator.

Recorder.

Power supply and control.

1.6 GHz interrogator.

Cubic CR-100 ground sited transponders.

C. Employment:

High precision bomb scoring.

Quick data turn-around.

One-day preparation.

Unmanned ground transponders.

Base maintenance.

D. Principal Sources of Error: Same as CIRIS.

E. Accuracy Specifications:

Position: 5 feet.

Velocity: 0.5 feet/sec.

F. Cost Estimates:

Pod: $350,000.
Transponders: $12,000 each.
Ground data terminal: $50,000.
Support equipment: $30,000.

G. Availability: Operating at Eglin AFB. Can be purchased.

IX. PATS, Sylvania (laser radar; azimuth, elevation, range)

A. Data Source/References:

1. Trip report.
2. Brochure.

B. Operational Description:

YAG, 1.06 micron wavelength.

Tracking laser.

Elevation over azimuth mount, ground based.

Retroreflector fastened to target.

Joystick acquisition.

Video camera co-mounted with laser for aid in acquisition.

Minicomputer.

Video recorder.

X-Y plotter.

Range counter.

Logic control unit.

Instrument van.

C. Employment:

Tracks mortar shell, helicopter, aircraft.

Maximum range, 100,000 feet.

Data rate: 10, 20, 50, 100/sec.

Coverage: Azimuth, ± 170 degrees; elevation, -5 to +85 degrees.

Slewing characteristics: 0.5 rad/sec, 0.08 rad/sec^2, azimuth and elevation.

Display: Range, 1-foot increments; elevation and azimuth, 1 degree increments.

Field of view: Video, 5 to 20 degrees (zoom); laser, acquisition, 3 millirad.

Set-up: 1 hour.

D. Principal Sources of Error:

Atmospheric refraction.

Optics mechanical error.

Servo and readout resolution.

E. Accuracy Specified (up to 65,000 feet):

Range, ± 2 feet.

Azimuth and elevation, 0.1 milliradian

F. Cost Estimate: $600,000, complete with instrument van.

G. Availability: Operational at Yuma Proving Ground.

X. A-7E Navigation and Weapon Delivery System (RRS, IMU)

A. Data Source/References: "A-7E - Simulation and Testing", 6th Guide Test Sympos [41].

B. Operational Description:

IMU: AN/ASN-90(V).

Doppler Radar Set (DRS): AN/APN-190(V).

Forward Looking Radar (FLR): AN/AFQ-126(V).

Air Data Computer (ADC): CP-953/AJQ.

Heads Up Display: AN/AVQ-7(V).

Projected Map Display: AN/ASN-99.

Tactical Computer Set (TC-2): AN/ASN-91(V).

C. Employment: Used on A-7E attack Naval aircraft. Paper describes lab simulation facility.

D. Principal Sources of Error: Not discussed.

E. Accuracy Specifications:

1. Probably not stringent because missiles require only rough aiming if they are homing devices.
2. Gun aiming probably uses feedback, miss distance error signal.

F. Cost Estimate: Not given.

G. Availability: All equipment in military arsenal.

XI. ACRS (Air Combat Maneuvering Range) Cubic (RRS, IMU, ground based data reduction and graphics display)

A. Data Sources/References: Trip report.

B. Operational Description:

Strapdown IMU.

Six ground based transponders.

Telemetry, Yuma Marine Air Station to Miramar Naval Air Station, San Diego.

Data reduction, recording at Miramar.

Graphics display on large screen CRT, with variable aspect, terrain, dynamics of encounter, scoring, time, printout availability.

Airborne equipment in sidewinder pod.

C. Employment:

Real (mock) dogfight recording, instant debriefing, detailed analysis of combat (32 reasons for a miss are available). Graphics from cockpit of "friend" or "foe", or any point external to action.

D. Principal Sources of Error: Same as for item VIII, ARIS.

E. Accuracy Specifications: Not given, probably same as item VIII, ARIS.

F. Cost Estimate:

Ground based transponder: $65,000 to $80,000 each.

Pod: $350,000.

Ground equipment: $1,500,000.

G. Availability: Operational at Air Marine Station, Yuma, and Miramar Naval Air Station, San Diego.

APPENDIX E

MILLIMETER RADARS

Table E-I lists the known (as of June 1974) U. S. radars in the frequency region 70 to 140 GHz (F. B. Dyer and E. K. Reedy, "Millimeter Wave Radars," 1974 IEEE S-MTT International Microwave Symposium Proceedings Georgia Institute of Technology, Atlanta, Georgia, June, 1974). Georgia Tech developed and fabricated prototypes of five of the radars listed.

The analysis in Chapter 3 indicated that a 70 GHz or 95 GHz radar mounted in an airborne pod with the components of a CIRIS-type reference platform locating system would permit the measurement of performance of low-flying missiles within the accuracy required by WSMR.

Prototype 95 GHz Radar

Georgia Tech has developed a number of millimeter radars. These will be described here to indicate the state-of-the-art. The first to be described is an instrumented, calibrated short pulse measurement radar operating at approximately 95 GHz. Major parameters of this radar are summarized in Table E-II. It is housed in a small, protective container which consists of two separate compartments; one containing the magnetron and modulator, shown in Figure E-1. The receiver is behind the antenna shown in Figure E-1. The packaging approach combines the desirable level of isolation of the functions needed to minimize interaction and interference problems with good portability and accessibility. Sufficient space was provided in the package to allow the radar to be used in a number of different experiments. The overall system configuration is shown in block diagram form in Figure E-2.

This radar could possibly be adapted for mounting in an airborne pod. The research and development effort would include modifications of the packaging to meet environment requirements. The R&D would also include the design and fabrication of a larger, steerable antenna, the means for acquiring and automatically tracking the transponder on the low-flying vehicle, the means for readout of angular directions and range to the target, and a compatible transponder. This seems to be a feasible, though difficult, electromechanical R&D task.

TABLE E-I

SURVEY OF KNOWN EXISTING MILLIMETER WAVE RADAR SYSTEMS

Identification	Application	Frequency	Sponsor & Source
AN/APQ-62	Side Looking Mapping Radar	70 GHz	-
JR-9	Search Mapping Radar	70 GHz	WPAFB & TRG Rayetheon
AN/BPS-8	Search Radar	70 GHz	-
AN/MPS-29	Search and Surveillance Radar	70 GHz	USAECOM & Georgia Institute of Technolog;
Experimental	Aircraft Obstacle Avoidance Aircraft Instrument Landing	70 GHz	USAECOM & Nordon Division, United Aircraft
Experimental	Obstacle Avoidance Sea Clutter Measurement	95 GHz	NADC, Warminster, Pa.
Experimental	Space Object Identification	95 GHz	Aerospace Corp., El Segundo, Cal.
Experimental	Arctic Terrain Avoidance	95 GHz	Navy & Applied Physics Lab, Silver Springs, Md.
Experimental	Airborne Applications; Instrument Landing, Short Range Weapon Delivery, Sensor Cueing	95 GHz	WPAFB & Goodyear Aerospace Corp. Litchfield Park, Ar.
Experimental	Low Altitude Aircraft Tracking; Target Acquisition, Basic Millimeter Wave Radar Studies	70 GHz	Ballistic Research Labs, Aberdeen Proving Grounds, Md.
Experimental	Noise Modulated Radar for Clutter Suppression for FM/CW for High Range Resolution	94 GHz	" " "
Experimental	Bistatic CW Radar for Cross Section Measurements	140 GHz	" " "
Experimental Rapid Scan	Ranging, Target Acquisition, Command Fusing, Fire-Control, and Navigation	70 GHz	Harry Diamond Laboratories and Georgia Institute of Technology
Experimental	Instrumentation for Basic Millimeter Radar Studies; Backscatter Studies, etc.	95 GHz, and 70 GHz	Georgia Institute of Technology
Experimental	Monopulse Tracking Investigations	70 GHz	Norden Division, United Aircraft
Experimental	Arctic Terrain Avoidance	95 GHz	Applied Physics Lab and Georgia Institute of Technology

TABLE E-II

PARAMETERS OF GEORGIA TECH GT-M EXPERIMENTAL RADAR

Parameter	Description
Frequency	95 GHz (Nom)
Peak Power	6 kW
Pulse Width	50 ns or 10 ns
PRF	0-4000 pps
Antenna Type	Paraboloid (Cassegrain)
Azimuth Beamwidth	.70°
Elevation Beamwidth	.65°
Gain	47.1 dB
Polarization	H or V
IF Center Frequency	60 MHz or 160 MHz
IF Bandwidth	20 MHz or 100 MHz
IF Response	Logarithmic (linear available)
Noise Figure	15 dB
Dynamic Range	70 dB
Display Type	A-scope, B-scope, PPI
Dimensions:	
Cabinets	36 x 36 x 30 inches
Antenna Dish	12 inches diameter

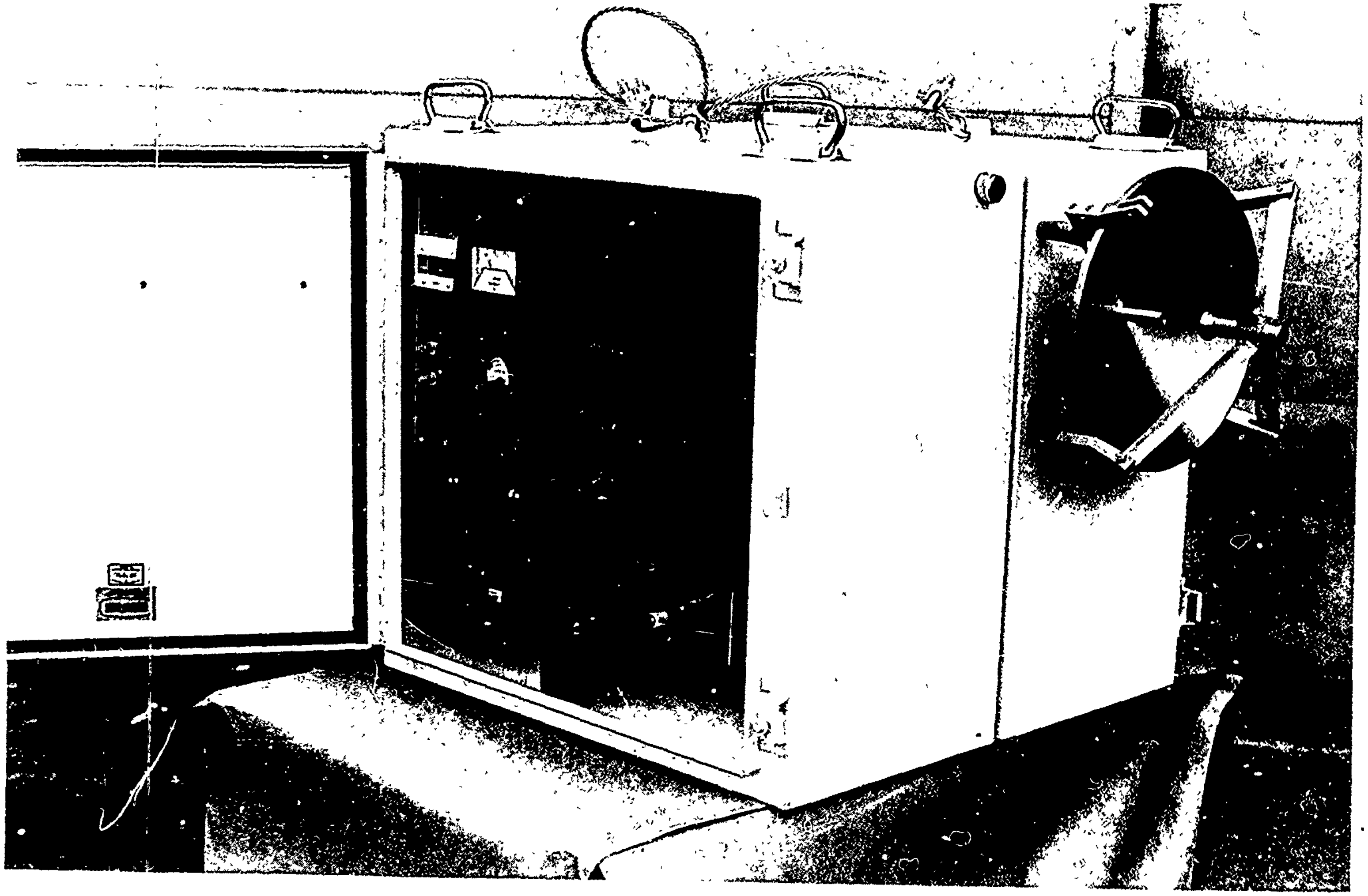

Figure E-1. Georgia Tech Experimental 95 GHz Radar, GT-M.

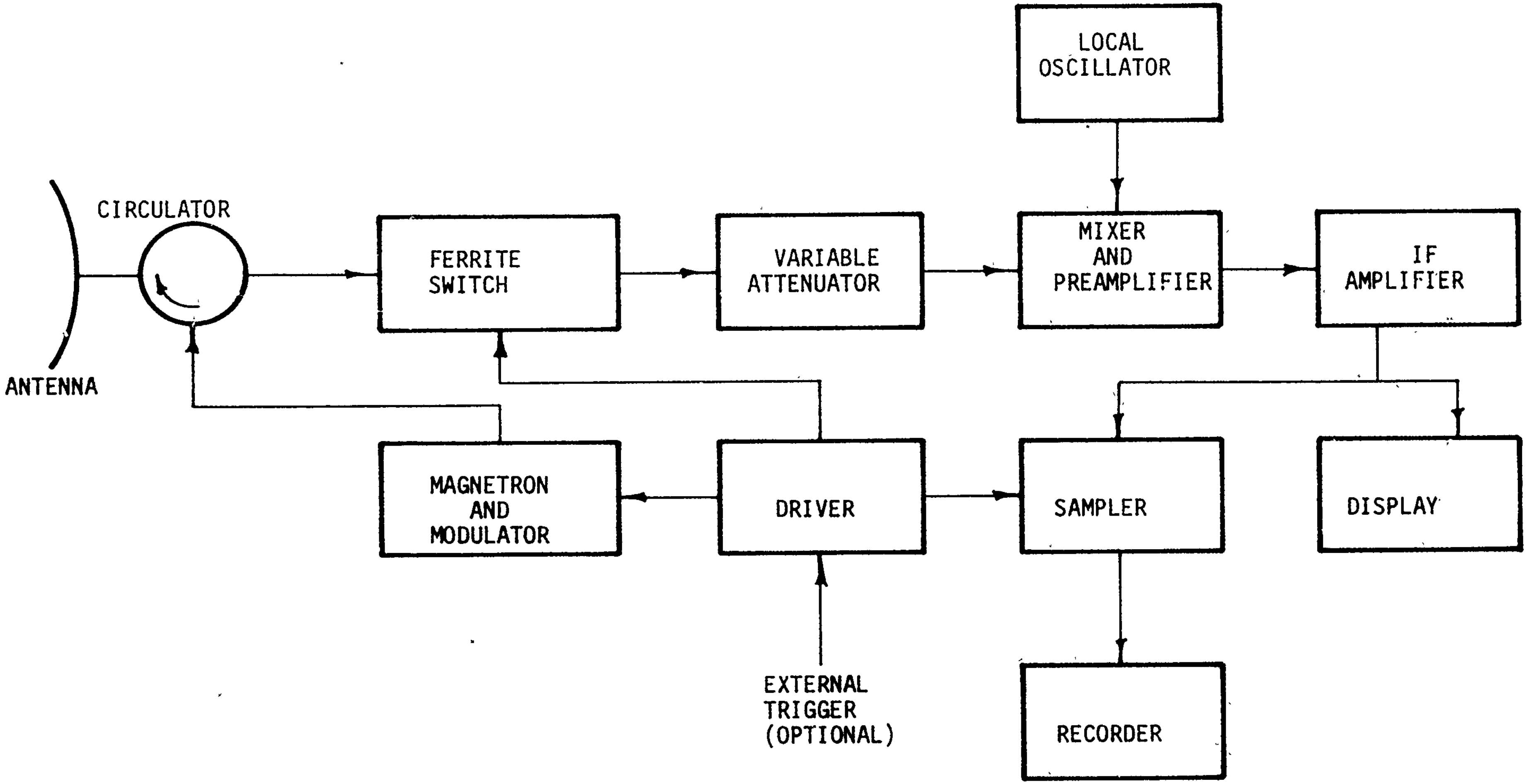

Figure E-2. Simplified block diagram of Georgia Tech 95 GHz radar, GT-M.

Several risk elements would have to be resolved before undertaking the development of the radar. They include the assessment of errors of the radome and the positioning and resolution errors if the antenna is to be a steered dish. Other risk elements would involve the method used in acquiring the target, and the automatic control loop used for tracking the target.

The beam of the antenna shown in Figure E-1 (0.7 degrees) is too broad because the dish is smaller than the one meter dimension found in this study to be required. One-fiftieth of 0.7 degrees is 0.24 milliradian, but our study has estimated the allowable resolution error to be 0.096 milliradian. The antenna diameter would thus have to be on the order of 2.5 times the diameter shown in Table E-II, or 30 inches.

The second Georgia Tech prototype 95 GHz radar system is described in Table E-III. The program under which this radar was developed required a fan-beam scanning antenna which is shown in Figure E-3. The thickness of the fan beam, 2 milliradians, is about 2.4 times smaller than the WSMR requirement we have estimated.

Prototype 70 GHz Radar

The AN/MPS-29 combat surveillance radar is a rapid-scan radar system designed, developed, tested, and evaluated by Georgia Tech for the U. S. Army Electronics Command during 1957-1960. The primary intent of this research effort was to develop and evaluate the performance of an experimental 70-GHz ground surveillance radar to provide high resolution display of ground targets at short ranges. A unique rapid-scan antenna was developed for this application. The scanning antenna for the AN/MPS-29 would be too wide (5 feet) for the WSMR requirement. It consisted of a geodesic Luneberg lens for azimuthal collimation and a modified parabolic cylinder for vertical collimation and beam shaping. The characteristics of the AN/MPS-29 are shown in Table E-IV and Table E-V.

A smaller version of the AN/MPS-29 was constructed, to mount in an armored personnel carrier. The smaller antenna is shown in Figure E-IV and described in Table E-VI.

TABLE E-III

CHARACTERISTICS OF THE SURFACE EFFECT VEHICLE
95 GHz SCANNING ANTENNA

Electrical	
Frequency range	93.0 - 97.0 GHz
Broad-plane beamwidth (E-plane)	1.5°
Narrow-plane beamwidth (H-plane)	0.11° (2 mrad) or less
Scanning in narrow-beam plane (H-plane)	± 1.0°
Sidelobe level	-20 dB wrt main beam
Polarization	Linear (in non-scan plane)
Power	6.0 kW peak
Gain	48.0 dB
Environmental	
Wind Velocity	20 mph
Temperature Range	-20 to +80°F
Scanner	
Scan speed	0 to 50 scans/sec continuously variable
Scanner position readout accuracy	0.1 mrad
Prime power	115 volt, 5 amp single phase AC, 60 Hz
Mechanical	
Resonant frequency	15 Hz minimum; vertical or horizontal mounting
Weight w/o transmitter or adapter	575 lbs.
G-loading	3 G max

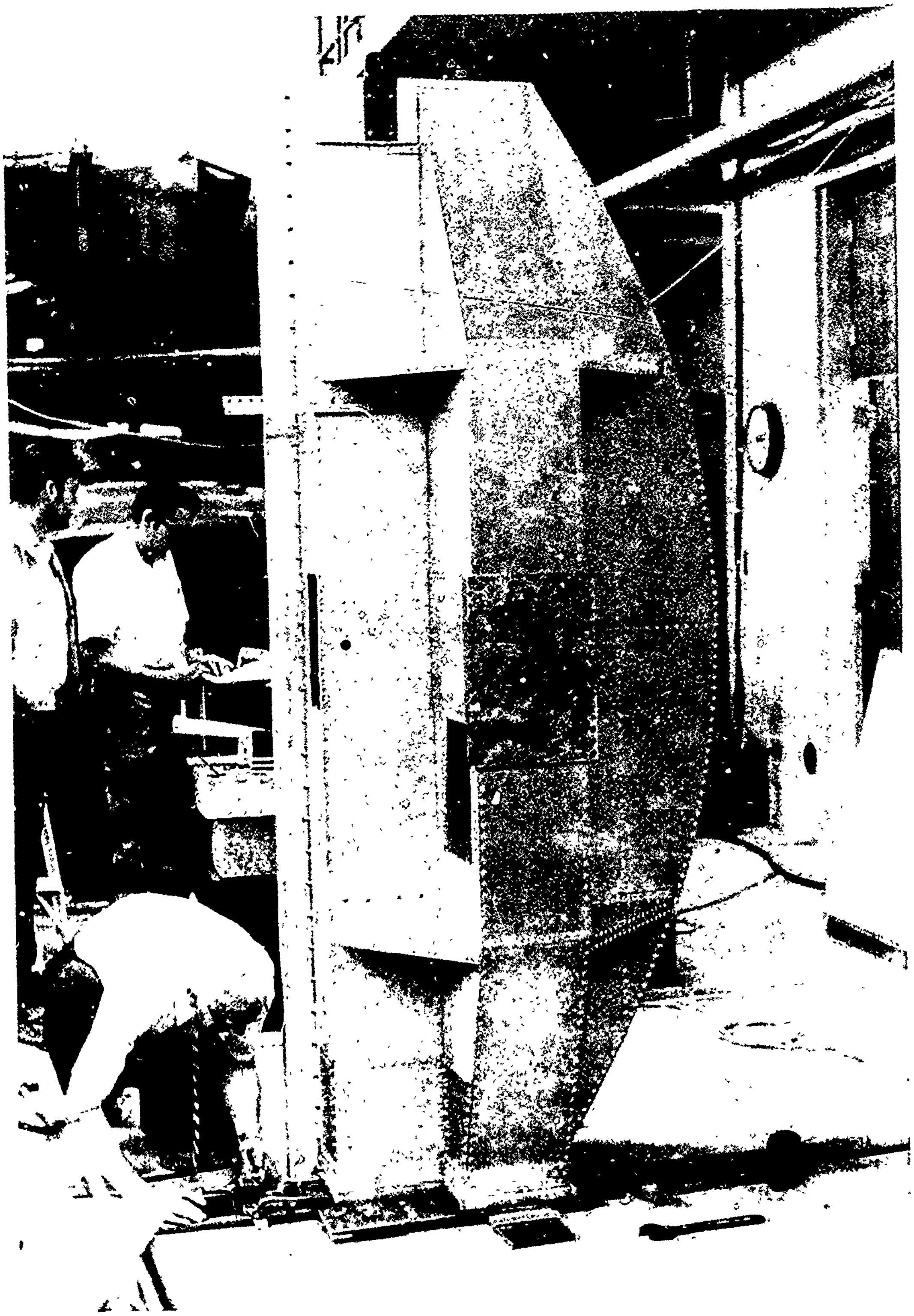

Figure E-3. Fan Beam, 95 GHz Radar Antenna, Designed and Fabricated by Georgia Tech.

TABLE E-IV

SYSTEM PARAMETERS FOR THE AN/MPS-29

Frequency	70 GHz
Azimuth Beamwidth	0.2° (3.5 mils)
Elevation Beamwidth	0.3° (shaped to -4° of elevation)
Pulse Width	0.05 μsec (7.5 meters)
Polarization	Vertical
Scan Rate	20 Scans per second
Scan Sector (Azimuth)	30° (150 Beamwidths)
PRF	10,000 pps
Antenna Gain	54.7 dB
Transmitter Power	15 kW
Receiver Noise Figure	18 dB
Doppler	Noncoherent

TABLE E-V

MAXIMUM RANGE FOR DETECTION OF TARGETS WITH AN/MPS-29

	B-Scope-Display
Walking man	5 km
Light vehicles	10 - 15 km
2-1/2 ton truck	18 km
Helicopter (H-19)	15 km
	Aural Display
Walking man	8 km
8 walking men	10 km

Figure E-4. Rapid Scan, Combat Surveillance Radar Antenna, AN/MPS-29, Designed and Developed by Georgia Tech.

TABLE E-VI

ANTENNA PARAMETERS FOR THE ARMORED PERSONNEL CARRIER'S FOLDED GEODESIC LUNEBERG LENS ANTENNA

Operating Frequency	70 GHz
Azimuth Beamwidth	0.55°
Elevation Beamwidth	Shaped (Positionable in elevation from -10° to 20°)
Polarization	Vertical
Scan Rate	1 Scan/Minute (Min) 70 Scans/Sec (Max)
Scan Sector (Azimuth)	45° (± 22 1/2° about boresight). Boresight may be varied over 360° of azimuth.
Antenna Gain (including losses)	43.2 dB
Dimensions	24-inch diameter by 3.5-inch height